

Letter from A.N. Prior to Alan Ross Anderson dated July 10, 1955¹

Canterbury College

Christchurch, N.Z.

10/7/55

Dear Dr. Anderson.

Many thanks for your letter of the 28th. I will be very glad of anything you can do when you can do it.

I must apologise for my error about the restriction on the first numeral in the value-sequences (it was not a case of hasty reading on your part but of hasty writing on mine) – the first numeral must be, not 1 or 2, but 1 or 3!

It might be useful to explain that in my lectures I use the system Q as a formalisation not only of modal logic but of what I call tense-logic, in which p, q, r &c. stand for ‘statements’ in a sense in which the same statement may true at one time & false at another time, & at another time perhaps not even capable of being made. The sequences can then be regarded as representing the truth values (in the ordinary sense: plus ‘unstatable’) of the statement a different times in an infinite collection of times, the first number is the sequence representing the present time. If the ‘unstatable’ possibility is left out of account, so that our sequences contains only 1’s and & 2’s², & the matrix-determinations are as before but with the reference to 2’s deleted, we get on the modal interpretation S5, & on the other interpretation as tense-logic isomorphous with S5. And the latter can be represented as a derived fragment in a system using the form Utp for ‘p at t’, with Mp defined as $\Sigma t Utp$ & Lp as $\Pi t Utp$, & with the following postulates :-

- (1). The rules and axioms of the classical prop. calc.³; & the Lukasiewicz’ rules for quantifiers
- (2). The special axioms
 1. CUtNpNUtp
 2. CNUtpUtNp
 3. CUtCpqCUtpUtq

¹ This is a transcription of a handwritten letter kept in the Prior Collection at Bodleian Library, Box 1, Oxford. Only the first page of the letter is deposited in the collection. It has been edited by Adriane Rini, Peter Øhrstrøm, Max Cresswell and David Jakobsen.

² Editors’ note: This should probably have been contains only 1’s and & 2’s.

³ Editors’ note: ‘prop. calc.’ is short for ‘Propositional calculus’.

4. $\text{C}\Pi\text{tUtp}$

5. $\text{CUt}'\text{UtpUtp}$

(3) The special rule RU: $\alpha \rightarrow \text{Ut}\alpha$

With the given definitions of M & L, the derivation of Gödel's postulates for S5 within the system is quite simple. A Ut-system with Q instead of S5 as its L-M fragment would have to drop axiom 2 & the rule RU; it could keep the rest, and I doubt whether the next would be sufficient without replacement for the terms dropped. (The intuitive justification for dropping 2 is easy to see when p is unstable, so is Np & so is 'Np at t', although it can be said now that p is unstable at t & therefore not true at t, so in this case we will have NUtp, 'Not p-at-t', but not UtNp, 'Not-p at t').

I have also another sort tense-logic which has S4 as its