Letter from A.N. Prior to Alan Ross Anderson dated June 21, 1955[1]

Canterbury University College

Christchurch N.Z

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Dear Dr. Anderson,

I hope you don't mind my writing to you out of the blue like this, but you were mentioned to me by Professor Church as someone who might be able to solve a problem in modal logic which interests me.

I am wanting a set of postulates for a system with an infinite characteristic matrix which can be intuitively approached as follows: If we make the following assumptions

(1) The ordinary propositional calculus holds in any possible state of affairs;

(2) Np is true in any possible state of affairs if and only if p is not true in that p.s.a.

(3) Kpq is true in any p.s.a. if and only if p is true in that p.s.a. and q is true in it.

(4)[3] Lp is true in a p.s.a. if and only if it is true in <u>every</u> p.s.a. <u>that</u> p is true in the p.s.a. in question,

the modal system we'll get is S5. The modal system I'm after, which I'll call Q, results from modifying these assumptions by recognizing that in some possible states of affairs a given statement p may be simply not statable at all. (Suppose, e.g., that there are logical proper names, p [p. 2] contains one, and in some possible states of affairs the object named doesn't exist).

If there were only two possible states of affairs A and B, A being the actual states of affairs, then propositions statable in A may be of any of the following 6 sorts: -

1. True in both A and B
2. True in A but unstatable in B
3. " " " and false in B
4. False in A and true in B
5. " " " and unstatable in B
6. " " both A and B.

Common sense then suggests the following tables for K, N, M and L (Mp = 'It is the case in some p.s.a. that p', Lp = 'it is the case in every p.s.a. that p'):-

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[1] This is a transcription of a handwritten letter kept in box 1 at the Prior Collection at Bodleian Library, Oxford. It has been edited by Max Cresswell, David Jakobsen, Adriane Rini and Peter Øhrstrøm.

[2] Editors' note: Prior has written 5 and crossed it over with 6.

[3] Editors' note: There is in the MS an marginal arrow pointing, not obviously by the same author. It might have been Anderson noting that this is the important clause.

K	1	2	3	4	5	6	N	M	L	NMN
*1	1	2	3	4	5	6	6	1	1	1
*2	2	2	2	5	5	5	5	2	5	2
3	3	2	3	6	5	6	4	1	6	6
4	4	5	6	4	5	6	3	1	6	6
5	5	5	5	5	5	5	2	5	5	5
6	6	5	6	6	5	6	1	6	6	6

The values 1 and 2 are designated because a formula counts as a law if all substitutions in it are true in all worlds in which they are statable. Note the difference between NMN and L. If p has the value 2, i.e. is true in A but unstatable in B, then it is not true in all worlds, and so Lp is false in A, but is unstatable in B because p is, so has the value 5. First [p. 3] under the same conditions – p true in A and unstatable in B – the statement that not-p is not true in any p.w. is <u>true</u> in A, though still stateable in B, so NMNp has the value 2.[4]

Consequential tables of C, A and E (with the usual definitions) are

C	1	2	3	4	5	6
*1	1	2	3	4	5	6
*2	2	2	2	5	5	5
3	1	2	1	4	5	4
4	1	2	3	1	2	3
5	2	2	2	2	2	2
6	1	2	1	1	2	1

A	1	2	3	4	5	6
*1	1	2	1	1	2	1
*2	2	2	2	2	2	2
3	1	2	3	1	2	3
4	1	2	1	4	5	4
5	2	2	2	5	5	5
6	1	2	3	4	5	6

E	1	2	3	4	5	6
*1	1	2	3	4	5	6
*2	2	2	2	5	5	5
3	3	5	1	6	5	4
4	4	5	6	1	2	3
5	5	5	5	2	2	2
6	6	5	4	3	2	1

These tables verify all formulae of Q, and some others arising from the fiction that there are only 2 possible states of affairs. The characteristic matrix for Q would be one with an infinite number of elements, with the following features: -

(1) Each of the elements may be associated with an infinite sequence of numbers, which must be 1,2 or 3, and of which the first must be 1 or 2.[5] Intuitively the number one amounts to 'is true in', 2 to 'is unstatable in' and 3 to 'is false in', and the p.s.a. concerned is indicated by the place of the number in the sequence, the first place representing the actual state of affairs.

(2) The sequence for Np is determined by the sequence for p as follows: -

N(xyz…) : (Nx)(Ny)(Nz)…., [p.4]

where the table for N considered as operating on a single number is

N	
1	3
2	2
3	1

[4] Editors' note: We are finding it difficult to make a transcription of this that makes sense to us.
[5] Editors' note: There is a circle around the number 2, and beside it, maybe in Andersons handwriting, is written 3.

(3) The sequence for Kpq is determined by the sequences for p and q as follows:-

K(xyz….) (x'y'z'.....,)

= (Kxx')(Kyy')K(zz')…..

Where the table for K considered as operating on a single pair of numbers

K	1	2	3
1	1	2	3
2	2	2	2
3	3	2	3

(4). The sequence for Lp is determined by the sequence for p as follows:-

(a) If that for p contains 1's only, so does that for Lp

(b) If that for p contains any 2's, then in that for Lp these 2's keep their place unaltered, and all other places are occupied by 3's

(c) If that for p contains no 2's, but contains 3's, whether it also contains 1's or not, that for Lp contains 3's only.

(5) The sequence for Mp is determined by the sequence for p as follows

(a) If that for p contains either 3's only or 2's only[6], that for Mp is the same as that for p,

(b) If that for p contains 2's and 1's whether it contains 3's or not, in that for Mp the 2's keep their place unaltered and all other places are occupied by 1's.

(c) If that for p contains no 2's, but does contain 1's, that for Mp contains 1s only.

(6) The designated sequences are all those which contain no 3's. [p. 5]

These possible states of affairs and whatnot needn't be taken too seriously metaphysically; they're just scaffolding to get the thing into a mathematically precise form. I don't know what sort of postulates would have this for a characteristic matrix (that's what I'm asking you!), but Q does have the following easily deducible relations between Q and other systems:-

(1) <u>Gödel's postulates for S5</u>

(a). The definition N = NLN cannot be used; CMpNLNp is verified, but not CNLNpMp

(b). The axioms CLpp and CLCpqCLpLq are verified, but not CNLpLNLp (CMLpLp is, however).

(c). The rule $\alpha \rightarrow L\alpha$ is not verified, though $\alpha \rightarrow NMN\alpha$ is.

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[6] Editors' note: In the margin, between 2's and only there is a question mark and an arrow taking a comment to the word only, which reads: 'Must mean stays same'. The handwriting is not Prior's, and quite likely is Anderson's.

(d). Classical propositional calculus verified.

(2) VonWright's system M''

(a) Two definitions L = NMN cannot be used; CLpNMNp is verified, but not CNMNpLp

(b) The axioms CpMp and CMNMpNMp are verified, though not the theorems CMpLMp), but not EMApqAMpMq (we have CMApqAMpMq, but not CAMpMqMApq)

(c) Neither the rule α → Lα (see Gödel (c)) nor Eαβ → EMαMβ is verified.

(d) As under Gödel.

Q is contained in S5, independent of S1-4 and of Feys' T, and inconsistent with S0-8 and with the Ł - modal system. [p. 6]

If we use the form Σφφx, i.e. 'There are facts about x' for 'x exists', it is of some interest that NMNΣφφx is a theorem but LΣφφx is not. Roughly, there is no possible state of affairs in which 'x doesn't exist' would be true; but 'x exists' needn't be true in-all-states-of-affairs.

That's the modal logic; now a bit of the biographical background. I have been invited to give the John locke Lectures at Oxford next session (expect to arrive there late January – leaving here end of December), and will be talking about 'Modality, Quantification and Time'. System Q comes in primarily as a tense-logic rather than a modal logic, with p,q,r, etc. for statements which might be true at one time and false at another, Mp for 'it is, has been or will be the case that p' and Lp for 'It is, always has been and always will be the case that p'. I play around with various systems for this, and with various ways of meeting difficulties about existence and the like. But one thing I just haven't the equipment for is finding postulate-sets to fit matrices and, would complete the thing very nicely if I could give a set for Q, with acknowledgements to the finder. I therefore wrote to Church a while ago asking if he knows anyone who might be interested in the problem and able to [p. 7] solve it, and he passed your name on as a possibility. If you did take it on I'd be very glad to send you a draft of the lectures once I have one typed. And if you made some interesting independent observations about Q in passing, it might be worth looking into the possibility of having them published as an appendix to the John Locke Lecturers when those are published, though it would probably turn out to be simpler for you to publish them separately somewhere.

From the latest Journal of Computing Systems it appears that we have a common interest in the Ł-modal system. There's a connection between my contribution and yours – if Δ is represented as a variable operator with the range restricted to S and V, the theorem AΔp∇q, with which your note is concerned, becomes rational enough. It expands by definition to AΔpCΔqq, and on my interpretation of Δ : similar substitution for it yields a formula in which one element is tautological.

Yours Sincerely

Arthur N. Prior