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THE LISTENER

Many-valued Logics

The last of three talks on 'The Logic Game' by A.N. PRIOR [1] [2]

Of this alone is Deity bereft,

To make undone whatever hath been done.

So wrote the Greek poet Agathon, and his words were quoted with approval by Aristotle in his *Ethics*. I had something to say in my last talk[3] about the analogies that may be drawn between the logic of necessity and possibility on the one hand and the logic of past, present and future on the other; but what these words of the poet point to is not just an analogy but a positive connection between these two things. What is necessary *par excellence*, in the sense of being beyond any possibility of alteration, is the past, 'whatever hath been done'. It is only with respect to the future that different possibilities are really open; with respect to the past we can say that this or that is 'possible' only in the sense that we do not know whether it was this or that that happened.

No one has yet produced a rigorous and satisfactory formal presentation of these connections between time and necessity; but the person who has come nearest to it is the late Professor Lukasiewicz,[4] and to have made a start on this is perhaps his most far-reaching achievement as a logician. What he put forward with this sort of problem in view was what he called a three-valued logic, that is a logic with three truth-values. This word 'truth-value' sounds alarmingly mathematical, but if is just a technical phrase designed to close up a rather obvious gap in the English language. When we ask whether something is heavy or light, we would be said to be asking about the weight of the object. Again, when we ask whether something is cheap or expensive, we would be said to be asking about its cost. Then

[1] Editors' Note: This text has been transcribed and edited by Jørgen Albretsen. The text is kept in the Bodleian Library, Oxford, Box 5. The page numbers in the original text have been put in curly brackets. In the Virtual Lab for Prior Studies, http://research.prior.aau.dk the text is stored under VL id 74.

[2] Editors' Note: With respect to the audio recording: Max Cresswell has with the kind help of Nga Taonga Sound & Vision - The New Zealand Archive of Film, Television and Sound, localised what very probably seems to be the only existing recording of Arthur Norman Prior's voice. PLEASE NOTE: only part of the recording is preserved. See footnote 5. http://www.ngataonga.org.nz/collections/catalogue/catalogue-item?record_id=168375 (starts at 15:17)

[3] In the original text, a footnote marked with a * at this point reads: "Printed in The Listener of April 25". It refers to the talk *Symbolism and Analogy: The second of three talks on 'The Logic Game' by A.N. Prior*, here found in VL id 73. VL id 72 contains *The Necessary and the Possible: The first of three talks on 'The Logic Game' by A.N. Prior*, The Listener, April 18 1957.

[4] Throughout the text, this spelling is used, not the correct: Łukasiewicz (starting with the 'L with stroke').

again, when we ask whether a man is tall or short, we would be said to be asking about his height. Now suppose we ask whether a statement is false or true – what are we asking about then? We have no ordinary word for it, so the logicians quite sensibly have coined one – they say we are asking about the statement's 'truth-value'. The truth-value of a statement is its truth or falsehood; at least, we ordinarily assume that those are the oniy two possibilities. Once you know this, it is obvious what a three-valued logic is; it is a logic in which allowance is made for statements being neither true nor false but some third thing of the same sort. And Lukasiewicz' view, when he first elaborated this system, seems to have been that statements about what has already come to pass, and what is bound to come to pass, are either true or false; but if is some still-future matter the outcome of which depends on chance or free choice, and the issue is not yet settled, then there are not as yet any facts one way or the other with which statements about this matter could agree or disagree, so we may say that such statements are neither true nor false but neuter, to give the third truth- value a name.

To explain one of the main results which Lukasiewicz achieve by means of these neuter statements I must introduce another {718} technical term which is not as alarming as it sounds, the term truth-function. In ordinary two-valucd logic we would say that a statement of the form 'p and q' is a truth-function of its two parts, meaning by this that whether the statement as a whole is true or false depends entirely on whether its parts are true or false. For example, if the separate statements 'grass is green' and 'the sky is blue' are both of them true, then the compound statement 'grass is green and the sky is blue' is automatically true too; and if either of the two separate parts is false, the compound is automatically false. We could set this out if we wanted to in a sort of multiplication table – in fact it will be exactly like a multiplication table for the numbers 1 and 0. A truth and a truth (that is a truth joined to a truth by 'and'), is a truth, just as 1 X 1 = 1; a truth and a falsehood is a falsehood, just as 1 X 0 = 0; a falsehood and a truth is a falsehood, just as 0 X 1 =0; and a falsehood and a falsehood is a falsehood, just as 0 X 0 = 0. Again, whether the statement 'grass is not green' is true or false depends entirely on whether the simple 'grass is green' is true or false. If 'grass is green' is true, 'grass is not green' is automatically false, and if 'grass is green' is false, 'grass is not green' is automatically true. So here we have another truth-function, and once again we could express its properties by means of a table, this time like the table for 1-x where x is 0 or 1. 'Not' applied to a truth gives a falsehood, just as 1-1=0, and 'not' applied to a falsehood gives a truth, just as 1-0=1.

Even the chief of all the logician's special words, the word 'if', has a sense in which it is truth-functional, though it is not often used in this sense in common speech. You can see what an advantage it is to be dealing just with truth-functions. Suppose you have a long formula, with lots of p's and q's in it that can stand for any statements at all, and you want to know whether this formula expresses a logical law, that is, you want to know whether it is true regardless of what the p's and q's stand for. If there is nothing in it but truth-functions, all you need to do is consider all the possible ways in which truth and falsehood can be distributed among your p's and q's, and work out by your tables what the whole thing comes to in each case; if it always comes out true, what you have is a law, and if it does not, it is not.

Expression of a Law

Let us take a simple example – the formula 'not both-p and not-p'. I have not given you a symbol for 'both' , it is actually K, so this formula works out as NKpNp. Suppose, first, that p is true. Then 'not p', by the table for 'not' will be false, and 'both p and not p' will combine a true and a false statement; and this, by the table for 'and', will be false. And since 'both p and not p' is false, 'not both p and not p' is true, by the table for 'not'. All that is on the supposition that p is true; but if you suppose p false the tables will give you the same result by a different route; so the thing is true either way – that is, this formula NKpNp expresses a logical law.

This method of verifying and falsifying formulae by calculation on the basis of truth-value tables goes back to C.S. Pierce, in a paper of 1885, and it is now, almost everywhere, one of the first things that students of logic learn. It came to this country, I think, through Wittgenstein, who started this unfortunate practice of describing anything verifiable by truth-value calculation as a 'tautology'.

There seem to be branches of logic in which truth-value calculation is not enough, and the logic of necessity and possibility is one of them. For it would seem that whether 'necessarily p' and 'possibly p' are true or false, does not depend solely on whether the simple p is true or false. It does, of course, depend on it up to a point; if the plain p is true, we know that 'possibly p' is also true, and if the plain p is false, we know that 'necessarily p' is also false. But if p is true, that is not enough to decide whether 'necessarily p' is true – some truths, we would ordinarily say, are necessary truths while others are merely contingent, that is, although they happen to be true they might very well not have been. And if p is false, that is not enough to decide whether 'possibly p' is false – some falsehoods, certainly, are also impossibilities but not all of them.

What Lukasiewicz argued was that it only seems impossible to represent the forms 'necessary p' and 'possibly p' as truth-functions because we assume that the only truth-values are truth and falsehood; if we admit neuter statements, it is a different story. Whether the forms 'necessarily p' and 'possibly p' are true or false is indeed not fully determined by whether the plain p is true or false; but whether 'necessarily p' and 'possibly p' are true, false or neuter is fully determined by whether the plain p is true, false or neuter.[5] It works like this: 'necessarily p' is automatically true if the plain p true, and false if p is false or neuter; and 'possibly p' is true if p is true or neuter, and false if p is false. This does not sound right; but it is not so bad if you remember the relation between necessity and time with which I started, and then remember what the three truth-values are. A statement is true only if it is a correct account either of something which has already come to pass, and so cannot any longer not have come to pass, or of something which has yet to come to pass but is already bound to do so. So only what is necessary for one reason or the other is really true either because it has already happened and cannot be altered or because it is bound to happen. And a statement is false only if it is an incorrect account either of something which has already come to pass, and therefore cannot now be as the statement said, or of something which has yet to come to pass but is bound to do so otherwise than as the statement says. So only what is impossible for one reason or the other is really false.

The True and the Necessary

This does not mean that whatever is possible is true, for although the possible comprises nothing that is false, it does comprise, beside what is true, what is not as yet either false or true. The fact that the true

[5] [Start of sound recording]. Prior makes several additions and alterations when reading the article text, to explain details further and surely to make it fit better to a talk over the radio.

and the necessary are co-extensive does not mean that the man who says 'My horse is bound to win' is never worse off than the man who says simply 'My horse will win'; for if the possibility of his horse's winning and the possibility of its not winning arc both still open, the man who says 'My horse is bound to win' has said something definitely false, while the man who just says 'My horse will win' has indeed said something which is not true, but it is not false either.

When looked at in this way, this account of the necessary and the possible is attractive; the real difficulties of three-valued logic arise when we consider words like 'and'. On ordinary two-valued assumptions, no logical word is more obviously and flatly truth-functional than 'and' but it is difficult to preserve this truth-functional character of 'and' in a three-valued logic; that is, it is difficult to maintain that whether the compound 'p and q' is true, false or neuter depends solely on whether its two parts are true, false or neuter. For when the two parts are both of them neuter, we want to say in some cases that the combination of them is neuter also, and in other cases definitely false. Suppose the two parts are 'John will wear a blue tie' and 'James will wear a red handkerchief' and both James and John have still to make up their minds on these matters; we are in this case happy enough about saying that the compound statement 'John will wear a blue tie *and* James will wear a red handkerchief ' is neuter in truth-value just as its parts are. But suppose the two parts are 'John will wear a blue tie' and 'John will not wear a blue tie'. If John has not yet decided whether to wear a blue tie or not, we again have a pair of neuter propositions, but the combination 'John both will and will not wear a blue tie' doesn't seem at all neuter – most of us would want to rule that out as plain false, and we would have no qualms in this case about equating the false with the impossible.

The Three-valued 'And'

Lukasiewicz took this dilemma squarely by the horns, and ruled that the three-valued 'and' is truth-functional, and that the result of combining two neuter statements by means of 'and' is always another neuter statement, even if it happens to be of the form 'p and not-p'. In this way he did give a clear and rigorous formulation to a logical system allowing for a third truth-value; but one cannot help feeling that he only did it by putting at least one poor word to something like forced labour, and I do not think we have yet heard the end of this story.

Lukasiewicz went on from the study of three-valued logic to the elaboration of similar systems involving not merely one but an indefinite number of truth-values in between the usual two; though in the closing years of his life he was for some reason strongly attracted to the view that the number of truth-values is precisely four. This last development is not one in which I am personally able to follow him;[6] my own view is that the future of the subject lies in the direction of the system with an infinity of values. It also seems to me that the notion of a truth-value needs to be broadened. With some sorts of logical calculation, what is important about a statement is not its simple truth or falsity, but when it is true, or in what possible circumstances it would be true, and it is sometimes useful to describe as a 'truth-value' the property of being true until five past six last night and never true {719} thereafter. In this area, as in others, the logician needs to have a nose for analogies; and what is important to him is the set of alternative possibilities we must reckon with when performing a given sort of calculation –

[6] One major omission by Prior in his radio talk, though, at this point, where the part of the text about precisely four truth values is left out.

that is what decides whether a system is two-valued or many-valued, in the most recent use of these terms.

I will finish with a little piece of history. As long ago as 1885, in the paper in which the idea of calculating with truth-values was first sketched, C.S. Peirce wrote this: 'According to ordinary logic, a proposition is either true or false, and no further distinction is recognised. This is the descriptive conception, as the geometers say; the metric conception would be that every proposition is more or less false, and that the question is one of amount'. Truth, on such a view, is zero falsehood, and it might have been better to talk about 'falsehood-values' instead of 'truth-values'. But Peirce only threw this out in passing as a possible line of development; it was left to Lukasiewicz and his school to make something of it.

-Third Programme