II. The Methods of Logic[1]

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{page 7} To show that a given inference is not valid, we need only find a "counter-example" to the form of inference which it exemplifies, i.e. an inference of exactly the same form which takes us from truth to falsehood. It is necessary to say "of exactly the same form" because a form of inference which is not itself valid may have special cases – sub-forms – which are valid, +[2] the given inference may exemplify one of these. For example "No A is a B, therefore no B is a C" is an invalid form of inference ("No horse is a dog, therefore no dog is an animal", which is of this form, would take us from a truth to a falsehood); but this does not mean that "No horse is a dog, therefore no dog is a horse" is an invalid inference, for the special sub-variety of the given form in which C is the same as A, i.e. the form "No A is a B, therefore no B is an A", is valid.

To show that a given inference or form of inference is valid is not in general so straightforward. For a number of limited but {page 8} important branches of logic, of which we shall give an example later, mechanical tests of validity – "decision procedures", as they are called – are available; but in others they are not. What is often done is to take a few forms of inference as "obviously" valid + to show that since these are valid, so are numerous other and more complicated forms in which the given forms are combined or transformed in various ways. An example may be given from the Stoic logicians. They assumed that we can safely take it for granted that, where P and Q are any statements, the following forms of inference are valid:-

I. If P then Q, but it is not the case that Q, so it is not the case that P.
II. It is not the case that both P and Q, but it is the case that P, so it is not the case that Q.

Now anything of the form

III. If both P and Q then R, but it is not the case that R, so it is not the case that both P and Q

would be justified by I, of which it is {page 9} a special case. So if we have these premisses

A. If both P and Q then R
B. It is not the case that R
C. It is the case that P

we can first infer "It is not the case the case that both P and Q" from A and B by III, and then combine this conclusion with C to obtain

D. It is not the case that Q

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[1] The text is kept in the Prior collection at Bodleian Library, Oxford. It has been edited by Martin Prior and David Jakobsen.
[2] Editor's note: Prior uses a '+' for 'and'.

by II. So we may use the validity of I and II to establish the validity of

IV. If both P and Q then R, but it is not the case that R, though it is the case that P, so it is not the case that Q.

In this way logical principles are themselves used to build up the principles of logic into large "deductive systems" rather like geometry or algebra.

Both in proving + disproving validity, it is obviously important to know what is the form of a given inference, and when one inference {page 10} is of the same form as another. This task is beset with two kinds of difficulty, one theoretical and one practical. In the first place, we have to decide which features or parts of inferences, + of the statements or propositions of which inferences are made up, contribute to their form, + which features or parts are non-formal + so capable of being replaced by schematic letters. Consider, for example, the inference

No stone is an animal
Therefore no stone is a dog.

Is this valid or not? It is not, if it is of exactly the same form as

No tree is a man
Therefore no tree is a plant

(which takes us from a truth to a falsehood); that is, if the most we can say about its form is that it is of the form "No X is a Y, therefore no X is a Z". But why not say it exemplifies the valid sub-form of this: "No X is an animal, therefore no X is a dog"? We can say this if we are prepared to count "animal" and "dog" as purely formal or logical words (words like "no" and "is" which are to be left in when we give the form of an inference). {page 11} All logicians would in fact say that "animal" and "dog" are not formal words, and that the given inference is not valid as it stands, but looks as if it might be because we all know that all dogs are animals, and if we state this explicitly the full inference has the form "All Z's are Y's, and no X is Y, therefore no X is a Z", which is valid. But how do we decide, in giving the form of an inference, which words to leave alone and which to replace by letters – which words are "formal" or "logical" and which words are not? This is in fact one of the most difficult and controverted questions in the philosophy of logic. There is general agreement that certain words, e.g. "No" and "is", are undoubtedly formal, and certain others, e.g. "dog" and "stone", are undoubtedly not so. But there are borderline cases about which there is no such agreement, e.g, in the inference

"I am sitting down, therefore it will always be the case that I have been sitting down", are "will" and "has been", i.e. words indicating tense[3] {page 12} part of the logical form or not? Some logicians say Yes, some No.

[3] Editor's note: If this text is from 1950, then this is perhaps the earliest discussion of the formal status of tense by Prior. It is perhaps more likely that the text, for the same reason, is later than 1950.

{page 13} Secondly, ordinary speech provides us with many alternative ways of exhibiting the logical features of a statement or inference, e.g. we have "All Xs are Ys", "Every X is a Y", "Any X is a Y", "Whatever is an X is a Y". Are these forms to count as logical forms or the same? Is, for example, "All dogs are animals" of the same form as "Whatever is a square is a quadrilateral", or are they of different forms? Most logicians would agree that these are differences that can be ignored, and that inferences which differ only in one having a premiss of the form "All Xs are Ys" and the other a premiss of the form "Whatever is an X is a Y" are to count as having the same form. And to save tediousness, and to develop their principles in a smooth and simple way, most logicians operate with a limited selection of forms, and leave it as a special exercise to figure out which of these forms best fits a given statement in ordinary language. They might say, for example, that "All dogs are animals" and "Whatever is a square is a quadrilateral", are both of the form "Every X is a Y". {page 14}

Here, too, there are controversial and borderline cases. What about "Only animals are dogs"? – is this just another way of saying "All dogs are animals", or does it exemplify a special form "Only Ys are Xs"? Some logicians find it worth while to introduce such a special form, and some do not; and some, again, will introduce it, but will define it as meaning the same as "All Xs are Ys" (and yet others might define "All Xs are Ys" as meaning the same as "Only Ys are Xs"). But whatever decision particular logicians make about particular cases, they all inevitably work with a limited number of forms, and makes generalisations about these forms in two main ways. In the first place, they give their standard forms technical names; for example they may say that any proposition of the form "Every X is a Y" is a universal affirmative proposition, {page 15} and if they have the form "Only Xs are Ys" they may call a proposition of this form an exclusive proposition. They can then use these technical terms to formulate rules of inference, for example that from a universal affirmative proposition we may validly infer the exclusive proposition in which the same terms are used but in reverse order, e.g. from "Every dog is an animal" we may validly infer "Only animals are dogs". Secondly, they may use symbolic abbreviation for their standard forms, e.g. they may write "Some Xs are Ys" as X:Y, and say that "X:Y, ∴ Y:X", is a valid form of inference. By this latter device, extremely complicated forms of inference can be set out in a compact way.

Advanced logical work is quite impossible without these schematic devices, but it is often a tricky matter to decide which of the logician's standard forms, if any, a particular argument in ordinary language really fits. For example, "Some mammals eat vegetables, therefore some vegetables eat mammals" might be thought to fit the form "Some Xs are Ys, therefore some Ys are Xs", {page 16} and to be a counter-example to that form. But if we want to express the premiss and conclusion of this inference in a standard way, the proper rendering of them would be "Some mammals are eaters of vegetables" and "Some vegetables are eaters-of-mammals", giving the inference the form "Some Xs are Ys, therefore some Zs are Ws", which is not valid at all. Or if we make use of a richer set of forms, we might regard the inference as being of the form "Some Xs are Ys of Zs, therefore some Zs are Ys of Xs", but this form isn't valid either (as the example shows), and isn't a sub-variety of the valid form "Some Xs are Ys, therefore some Ys are Xs".