III. The Branches of Logic[1]

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In some of the examples that we have so far used, we have put letters for common nouns like "dogs", "animals", etc., leaving the "form" to be carried by expressions like "No", "Every", "is" and "are", which go to form sentences from such nouns. The first inferences to be given systematic logical consideration, by Aristotle, were ones which thus depended for their validity on the logical behaviour of words like these. In other examples, we used schematic letters for entire sentences, the logical form being carried by expressions like "if", "and" and "It is not the case that", which form compound sentences from whole sentences. Inferences depending on the logical behaviour of expressions of this sort first systematically studied by the Stoics, and might be thought to belong to a more advanced branch of logic than the "Aristotelians" part of it. In modern systematizations of subject, however, the logic of propositions formed from other propositions is treated first, and other branches {page 18} of logic built upon it.

The study of forms of inference in which whole sentences are replaced by schematic letters is nowadays called the "propositional calculus", or sometimes the "sentential calculus". These terms are often further confined to the study of those compound sentences which are "truth functions" of their components. A compound sentence is said to be a truth-function of its compound sentence or sentences if the truth or falsehood of the whole depends solely on the truth or falsehood of its parts. For example, whether "It is not the case that grass is green" is true or false depend solely on whether grass is green is true or false; it is in fact false because "grass is green" is true. Similarly "It is not the case that grass is pink" is true because "grass is pink" is false. And in general, "It is not the case that P" is false if P is true and true if P is false.

{page 19} The subject of logic began as a study of various forms of inference or argument, with a view to determining which of them are valid or sound, and which are not.

In an inference or argument we pass from a certain statement or propositions, or from certain statements or propositions, called the premiss or premisses, to a statement or proposition called the conclusion. For example, in the inference

	(1)	Either the Head will come the Head's deputy will come;
but	(2)	The Head will not come;
so	(3)	The Head's deputy will come,

The propositions (1) and (2) are the premisses, and (3) is the conclusion. In the presentation of an inference or argument, the premisses are commonly stated first and the passage to the conclusion indicated by the word "so" or "therefore". But sometimes, as in the inference

	(4)	No Christians are Communists,
For	(5)	No Communists are Christians,

[1] The text is kept in the Prior collection at Bodleian Library, Oxford. It has been edited by Martin Prior and David Jakobsen.

the conclusion is stated first, and its derivation from the premiss or premisses indicated by the word “for” or “since” or “because”.

Inferences are of different forms. The {page 20}definition of the word “form”, or of the phrase logical form, is one of the hardest problems in the philosophy of logic; but (3) above is derived from (1) and (2) by an inference which would commonly be said to be of the same form as this one:-

	(6)	Either I planted peas in that row or I planted beans that row;
But	(7)	I did not plant peas in that row;
So	(8)	I planted beans in that row.

While (5) above is derived from (4) by an inference of the same form as this:-

	(9)	No eight-legged animals are insects,
For	(10)	No insects are eight-legged animals.

It is common to indicate the form of an inference by deleting all the words which make no difference to the form and replacing them by schematic letters. Thus the inference of (3) from (1) and (2), and that of (8) from (6) and (7), are both of the form

(11) Either P or Q, but not P, so Q;

While the inference of (4) from (5), and that of (9) from (10), are both of the form

(12) No A’s are B’s, for no B’s are A’s.

Forms of inference are divided into those which are valid (or sound, or safe) and those which are invalid (or unsound, or unsafe). A form of inference is valid if and only if no inference of that form could have all of its premisses true and its conclusion false. Both (11) and (12) above are valid forms of inference, but this:-

(13) Every X is a Y, so every Y is an X

is not. The simplest way to show the invalidity of a form of inference is to produce an actual inference of that form in which all the premisses are true and the conclusion false. For example, to show that {page 21}(13) is an invalid form of inference, it suffices to observe that

(14) Every horse is an animal, so every animal is a horse,

is of this form, and its one premiss is true but its conclusion is not. (14) would be said to be a counter-example to the inferential form (13).

Any actual inference which is of a valid form may be said to be a valid inference. For example the inferences of (3) from (1) from (2), of (4) from (5), of (8) from (6) and (7), and of (9) from (10), are all valid inferences. A valid inference, it should be noted, does not have to have true premisses or even a true conclusion. For example,

(15) All birds are breezes, and all breezes are bathing-machines, so all birds are bathing-machines,

though all the propositions in it are false, is a perfectly valid inference, being of the form

(16) All X's are Y's, and all Y's are Z's, so all X's are Z's,

which cannot have true premisses without the conclusion being true too; i.e. use of the form (16) cannot possibly lead us astray, though if we are astray to start with (as in (15)) it may leave us there.