Text 103: (Read Russell on Ag. And Dem.)[1]

A.N. Prior

Suppose we write

ΠxFx	for	"For any <u>cogitabile</u> x, Fx"
ΣxFx	for	"For some <u>cogitabile</u> x, Fx"
∀xFx	for	"For any real thing x, Fx"
∃xFx	"	" some " " ", Fx"

& define the latter two thus: --

$\forall x Fx = \Pi x\,(Rx \rightarrow Fx)$

$\exists x Fx = \Sigma x\,(Rx \,\&\, Fx)$

& Π and Σ have Lemmon's rules.
We then have

$\Pi x Fx \vdash \forall x Fx$
$\exists x Fx \vdash \Sigma x Fx$

$\Pi x Fx \vdash Fa$
$Fa \vdash \Sigma x Fx$

$\forall x Fx, Ra \vdash Fa$
$Fa, Ra \vdash \exists x Fx$

Now let's go back to our arg.

(1) $\Sigma x\,(Fx \,\&\, Rx)$
∴ (2) $\exists x Fx$

And Anselm gives an argument for (1), using

$I\exists x Fx \vdash \Sigma x Fx$

So by substitution

$I\exists x\,(Fx \,\&\, Rx) \vdash \Sigma x\,(Fx \,\&\, Rx)$.

Note also that

$\exists x Fx = \Sigma x\,(Fx \,\&\, Rx) = \Sigma x\,(Fx \,\&\, Rx \,\&\, Rx) = \exists x\,(Fx \,\&\, Rx)$

So

$I\exists x Fx = I\exists x\,(Fx \,\&\, Rx)$.

So now we have

$I\exists x Fx \rightarrow I\exists x\,(Fx \,\&\, Rx)$
$\rightarrow \Sigma x\,(Fx \,\&\, Rx)$
$\rightarrow \exists x Fx$

-- an awful result. We get it from <u>two</u> principles:-- {2}

[1] Edited by Max Cresswell and Adriane Rini. The original is kept in the Prior collection at Bodleian Library, Oxford, Box 6. The page numbers in the original has been put in {…}.

(1) $I\exists x Fx \rightarrow \Sigma x Fx$
(2) $\exists x Fx = \Sigma x\,(Fx \,\&\, Rx)$.

So better drop one of them. I think there are reasons for dropping (2), but I'll put them aside for the moment. What I want to say now is that even apart from ont. arg. there are reasons for being suspicious of (1). Let Fx be "x is the only thing there is".

So let our $I\exists x Fx$
be (4) $I\exists x \forall y\,(y = x)$.
"It is impossible that something is the only thing there is"
i.e. "It is imaginable that there should be only one real object."

Does this imply
(5) $\Sigma x \forall y\,(y = x)$
i.e. "Some *cogitabile* is the only real object"
Surely not. (5) means that there is some imaginable state of affairs such that [there?] *in that state of affairs* there is only one real object. (5) means that there is an imaginable object such that the only real object *in the actual state of affairs* is that object.

But the introduction of I raises the larger question of whether we really need these *cogitabilia* & *possibilia* at all. Can't we deal with the imaginable & the thinkable & the possible just by having *ordinary* quantifiers & operators {3} like I.? If we want to say, for example, that it can be imagined that some real thing is perfect, can't we just say that & be done with it. Note that
$I\exists x Fx$ ╪>[2] $\exists x I Fx$
"It is imaginable that something (real) is perfect" doesn't entail "there is some (real) thing of which it is imaginable that it is perfect". And this is still more obvious with more modest operators than "I". For example "I think that someone stole my pencil" ╪> "there is someone of whom I think that *he* stole my pencil." And "Dickens made up a story to the effect that someone was called Mr. P" ╪> "there was someone of whom D made up the story that he was called Mr. P." These are fallacies, but perhaps we have an urge to connect them, & then we check ourselves & say "well, there wasn't a *real* someone that D. said was called Mr. P., but there was an imaginary someone of whom he said that." But do we *have* to make this move? Does "There was an imaginary someone of whom D. said he was P" mean anything more than "D. said that there was a real someone who was P. – there *wasn't* someone who *was* P, there wasn't even anyone of whom D. said he was P, but D. *said that someone* was P." Isn't that all we want? {4} Well, we can certainly get quite a long way with this apparatus. Let's look at some old questions. I want to sum all this up by an historical puzzle & a modern answer to it.

When Aristotle uses the predicate 'exists' he seems to say inconsistent things about it. Sometimes he seems to accept
$Fa \vdash Ra$ (eg. Soc. is ill ⊢ Soc. exists)
& so
$-Ra \vdash -Fa$ (Soc. doesn't exist ⊢ Soc. isn't ill).
But elsewhere he comments that "X is Y" *doesn't* always imply "X is". He gives as an example "X is thought about" – "Mr. Pickwick is thought about" for example,

[2] The symbol used is a rightarrow with stroke through it. We represent it here as '╪>'.

doesn't imply "Mr. P. exists." Some medieval writers divide predicates into those that imply or presuppose existence & those that don't. What would Russell say? Or Lemmon. Well, the first point is that our X is Y might be of 2 forms. Either X is a log.prop.name, or it is a description. If it is a log.prop.name, either we say "a exists" is meaningless or we define it as exists Y (a = Y). If the latter we do have

Fa ├ Ra

i.e. Fa ├ ∃y (a = y)

But more than that, we have

├ Ra.

Indeed we could get this from

Fa ├ Ra.

Because -Fa is a case of Fa, so we would have

-Fa ├ Ra. {5}

So we now have

2	Fa ∨ -Fa	
2 (2)	Fa	A
2 (3)	Ra	2, Aristotle
4 (4)	–Fa	A
4 (5)	Ra	4, Aristotle
(6)	Ra	1, 2, 3, 4, 5, ∨E.

But suppose that the X in out 'X is Y' is a description, i.e. suppose the thing is

The thing that F's G's.

This =

∃1xFx & ∀x (Fx → Gx)[3]

& implies

∃1xFx

i.e. The thing that F's exists.

What about "The thing that F's <u>doesn't</u> G."?

Well this might mean

∃1xFx & ∀x (Fx → -Gx).

& this also implies

∃1xFx.

But we don't have ├ ∃1xFx because we don't have a as law

(∃1xFx & (∀x (Fx → Gx)) ∨ (∃xFx & ∀x (Fx → -Gx)).

Or "The thing that F's doesn't G" might mean –(∃1xFx & ∀x (Fx → Gx)).

& this doesn't imply

∃1xFx.

Russell dists. bwt. Primary & secondary occurrences of "The thing that F's". Secondary occurrence if it falls within the scope of some operator like -- .Also like "It is believed that". (<u>Extensional</u> or <u>intensional</u> complexity). {6}

Sometimes[?] distn. bwt. int. & ext. complexity if we allow empty names.

-Fa might be -(Fa) or (-F)a.

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[3] [Transcribers' note:] this seems to be the exactly one quantifier, and is written with the 1 immediately below the ∃.

$$\frac{Fa \vdash Ra}{(-F)a \vdash Ra}$$

But <u>not</u>

$$\frac{Fa \vdash Ra}{-(Fa) \vdash Ra}$$

In this case, $\dashv$ Ra.

Strawson treats descriptions like R. treats names.[4]

1,3 (8) ∃x (∀y (Py → Hxy) & Hxe)	7, E I
1,2 (9) ∃x (∀y (Py → Hxy) & Hxe)	2, 3, 8, E E

The usual move, confronted by this argument is to say that the first premiss is false, existence not being a perfection because it is not even a predicate. Well, here we haven't made it a predicate, but a subject; but that is almost certainly even worse, & there is a slight modification of the above argument in which "exists" does figure as a predicate. This modification makes the argument run as follows: --

(1) Whatever is perfect really exists
(2) Some thinkable[?] is perfect
(3) ∴ Some thinkable both is perfect & really exists.
(4) ∴ Something that is perfect really exists.

In symbols we could use P, T & R for "is perfect", "is thinkable" and "really exists", & give the proof as follows: --

1 (1) ∀x (Px → Rx)	A
2 (2) ∃x (Tx & Px)	A
3 (3) Ta & Pa	A
1 (4) Pa → Ra	1, U E
3 (5) Pa	3, & E
1, 3 (6) Ra	4, 5, MPP
1, 3 (7) Pa & Ra	5, 6, & I
1, 3 (8) ∃x (Px & Rx)	7, E I
1, 2 (9) ∃x (Px & Rx)	2, 3, 8 E E {3}

To this argument the contention that "exists" isn't a predicate would certainly be relevant, if it were true. But is it? It is certainly true that many ordinary assertions & denials of existence can be put into predicate calculus symbolism without using anything corresponding to our <u>R</u>. To use the stock examples, "Lions exist" comes out as <u>∃xLx</u> and "Unicorns do not exist" as –<u>∃xLx</u> . Here the grammatical subject, "lions' or "unicorns" has a predicate hidden in it ("is a

[4] [Transcribers' note:] the next few pages are not always consecutively numbered and it's not clear just how they relate to what has gone before. The first page is numbered {2}.

lion" or "is a unicorn"); this becomes the explicit predicate of the predicate-calculus translation, & no other predicate is needed. The "exists" bit has become absorbed into the quantifier, & the operator ∃x( )x is not a predicate, i.e. something that forms a sentence from a name, but is something that forms a sentence from a predicate.

But do we not sometimes use "exists" to form a predicate from a name, and moreover from a name that has no predicate hidden in it? What about "John Smith exists"? Well, perhaps "John Smith" does have a predicate concealed in it, namely the predicate "– is John Smith", i.e. "– is identical with John Smith." In that case, "John Smith exists" means "Something is John Smith", i.e. $\exists x\ (x = a)$, where a is John Smith. Now, however, there is in the symbolic version {6}[sic] something that corresponds to a genuine predicate, namely "Something is identical with –", which does form a sentence when attached to a name. We might in fact define the predicate "– (really) exists" by

$Ra = (\text{Df.})\ \exists x(x = a)$;

Or perhaps more naturally by the separated form

$Ra = (\text{Df.})\ \exists x(a = x)$,

i.e. "a exists" = "a is something", i.e. "There is something that a is", i.e. "There is something the a is identical with".

So perhaps there is a sense of "exists" in which it is a genuine predicate, & in which "John Smith exists", for example, is a genuine predication. But what about "Mr. Pickwick doesn't exist"? This, on the view under consideration, would have the form –Ra, i.e. $-\exists x\ (a = x)$, with a for Mr. Pickwick. Unfortunately in Lemmon's system, & in most normal systems of predicate calculus, this has contradictory consequences. Without going into the formalities, the argument is that Mr. Pickwick is after all Mr. Pickwick (by the Law of Identity) so there is an x such that Mr. Pickwich is x. Given the ordinary formal machinery, then, this definition {7} will not do for that sense of "exists" in which we want to say that some individuals exist & others (e.g. Mr. Pickwick) do not.

The answer to this may be to modify the "ordinary formal machinery" in some way; & various modifications of it have in fact been suggested. Let us stay with it for a while, however & try another tack. Let us keep the existing formalism but re-interpret it, so that ∃x means "For some x, real or imaginary", & then let us say that some of these real-or-imaginary objects are real, i.e. have the predicate R which we can leave undefined; & others of these real-or-imaginary objects lack this predicate. What becomes of the ontological argument then? It still seems to demonstrate that a real-or-imaginary object that has all perfections is in fact real, i.e. falls into the "real" compartment of this intended universe. And we cannot now deny the first premise on the ground that (real) existence isn't a predicate – in the system we are working, with it is. But what about the other premise, the one that asserts that some thinkable has all perfections, {8} including existence?

Anselm's ground for asserting this premiss was that even the man who denies the existence of a perfect being, if this denial is not mere verbiage, must have found it possible to imagine such a being – to entertain the thought that something is perfect, even if only to deny it. And to entertain that thought, & think it through, is to entertain the thought that something is perfect &

really exists (for nothing can be perfect without that). So the argument by which the second premiss is arrived at is this: It can be thought that something is perfect & real; so something real-or-imaginary (something thought of) is perfect & real. And the principle of it is: If it is imaginable that something is F, then some imaginable thing is F. If we write IP for "It is imaginable that P", the principle is

$$\underline{I\exists xFx} \rightarrow \underline{\exists xFx}$$

Note that on the right-hand side the $\exists \underline{x}$ still only means "For some real-or-imaginary x" – the principle doesn't enable us to pass straight from "It is imaginable that something real-or-imaginary is F" to "Something real is F". However, if the predicate "real" is {new page} a conjunct of F, we do get "Something real" on the right, & so the ontological argument goes through.[5]

So perhaps it is dangerous to have "real" as a predicate. But if we don't, how do we say "Some things are real & some things are not"? One suggestion is that we follow the current practice of putting existence into the quantifiers, but have two sorts of quantifiers, say $\underline{\exists(m)x}$ for "For some real or imaginary x" and $\underline{\exists(r)x}$ for "For some real x"

$\Sigma x - Rx$
$\rightarrow I\,\exists x(-Rx)$
$\rightarrow I\,\Sigma x\,(-Rx \,\&\, Rx)$. $\qquad I\,\exists x - Rx$
$\rightarrow I\,\Sigma x\,(-Rx \,\&\, Rx \,\&\, Rx)$
$\rightarrow I\,\exists x\,(-Rx \,\&\, Rx)$.

$I\,\exists xPx$
$\therefore \quad \Sigma xPx$
$\Pi x\,(Px \rightarrow \exists y\,(x = y))$
$\Pi x\,(Px \rightarrow (Px \,\&\, \exists y\,(x = y))$
$\therefore \quad \Sigma x\,(Px \,\&\, \exists y\,(x = y))$
$\therefore \quad I\,\exists x\,(Px \,\&\, \exists y\,(x = y))$. OK.

$\exists xFx$
[not=] $I\,\Sigma x\,(\exists y\,(x = y) \,\&\, Fx)$.

$\exists xFx$
$= \Sigma x\,(\exists y\,(x = y) \,\&\, Fx)$

$\Sigma x - \exists y\,(x = y)$
But $I\exists x\,\underline{-\exists y\,(x = y)}$.
$\rightarrow I\exists x - (x = x)$ No.

$\exists xFx$
$\exists x\exists y\,(x = y \,\&\, Fy)$.
$\rightarrow \exists y\,(x = y)$
[$\Sigma x\,Px \,\&\, \exists y\,(x = y) \longleftrightarrow \exists y\,(Px \,\&\, x = y)$.
? $\Sigma x\,(Px \,\&\, \exists y\,(x = y) \longleftrightarrow \Sigma x\exists y\,(Px \,\&\, x = y)$]

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[5] [Prior's note, in the margin:] $I\exists x\forall y\,(x = y) \rightarrow \Sigma x\,\forall y\,(x = y)$

$\exists x - \exists y\ (x = y)$

$\rightarrow \exists y\ ($

$[\exists x - \exists x\ (x = y)$

$\rightarrow \exists y\ (x = y\ \&\ - \exists x\ (x = y))$

$\exists y]$

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