The Modern-Traditional Logic of Keynes and Johnson[1]

by

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When I first studied logic, in the 1930's, my text-books were Stebbing's Modern Introduction to Logic and Mace's Principles of Logic; & those of us who wished to do further reading in the subject were encouraged to tackle two works to which both Mace & Stebbing were heavily indebted, Johnson's Logic & the Formal Logic of J.N. Keynes. Behind that lies a chapter in the history of logic which has yet to be written, & ought to be written. Keynes & Johnson were & are good books, even great books, and there is much in them which, either directly or through Stebbing & Mace & others, has become part of the almost unconscious background of 20th-century logical thought. The two books grew together, & the picture of their growth, which it is not difficult to build by studying the four successive editions of Keynes, & then Johnson, is a fascinating one.

[In one way the relation between Johnson and Keynes was very like that between Mill and Whately. Johnson's Logic, like Mill's, dealt extensively with induction, & on the deductive side was designed to be read 'in close connection with Dr. Keynes's classical work' {2} Mill's System was designed to be read in connection with Whately's Elements. But there was another relation between them. In his second]

The first edition of Keynes appeared in 1884 under the title Studies & Exercises in Formal Logic, including a Generalization of Logical Processes in their Application to Complex Inferences. Its final three parts dealt with Terms, Propositions & Syllogisms in a substantially 'traditional' manner, while Part IV was devoted to a study of conjunctive & disjunctive terms, being a little indebted to Boole & Schroeder & much more to de Morgan, Jevons & Venn. This arrangement was reproduced in the two following editions (1887 &1894), but the 'modern' material of Part IV came increasingly to affect, & to some extent to be absorbed in, the treatment of the traditional logic in Parts I to III, which in the 4th & final edition (1906) Part IV became simply a large appendix. Part I of Johnson's {3} Logic appeared 1921, Part II in ; Part III in . In one way the relation of Johnson to Keynes was like that of Mill to Whately at an earlier period – Johnson's Logic dealt extensively with induction, & on the deductive side was designed to be read 'in connection with Dr Keynes's classical work', as Mill's System was designed to be read in connection with Whately's Elements. But there was another side to their relationship. In all editions but the final, Keynes expressed his debt to the informed assistance of Johnson, & numerous footnotes indicate this indebtedness in detail; & apart from this extensive informal collaboration, the third (1894) edition owes much to Johnson's articles on 'the Logical Calculus' in Mind for 1892.

[1] Edited by Max Cresswell and Adriane Rini. The original is kept in the Prior collection at Bodleian Library, Oxford, Box 6. The page numbers in the original are put in curly braces.

§1. The Seven Relations between Propositions & between Classes – As a final illustration of the story of growth which I have mentioned, we must[?] take something which is now a commonplace matter – the theory of the 'seven logical relations'.

In Jevons's Principles of Science, Chapter VI we are told that 'any one proposition or group of propositions may be classed with respect to another proposition or group of propositions, as 1. Equivalent, 2. Impossible, 3. Consistent, 4. Contradictory, 5.' By way of illustration {4} 'all immortals are not men' is given as equivalent to 'all men are mortals', 'some mortals are men' as inferrible from it, but not equivalent, 'all not-men are not mortals' as consistent with it but not inferrible, & 'all men are immortals' as 'of course contradictory'. Keynes in the second edition of his Formal Logic has at the end of his treatment of immediate inferences a new section (§88) on 'Mutual Relations of Propositions', in which he refers to this passage from Jevons, & enlarges Jevons's four relations to five, by dividing what Jevons calls 'contradictions' into (a) contradictions in the ordinary sense, & (b) inconsistency that is not contradiction.

In the third edition this material is transferred to the chapter on opposition, as Section 57, though it is observed in a footnote that in this extension of the theory of logical relations between propositions some knowledge of immediate inferences is assumed; & the number of relations is enlarged to six – a pair of propositions may be (1) equivalent (as All S is P & All not-P is not-S); (2) one formally inferable from the other but not vice versa (as Some P is S & All S is P; (3) such that one must be true & both may be {5} (as Some S is P & Some not-S is P); (4) independent (as All S is P & All P is S); (5) contrary; & (6) contradictory. The new item here is (3), which corresponds to subcontrariety in the traditional square. It is clearly the transfer of the material from the discussion of immediate inference to that of opposition which have suggested the addition.

In the 4th edition (§84) the same possibilities are given, but with a re-numbering. Possibility (2) is now labeled '(2) or (3)', & the remaining numbers go up one; the new distinction is clearly that between the case in which the given proposition is implied by the other & the case in which it implies it. And this is connected with another development. In the third edition, towards the end of the chapter on 'the Diagrammatic Representation of Propositions' there is a new section (§93) in which Euler's five diagrams are enlarged to seven, his last to cases being each subdivided into the case in which the two classes exhaust the universe between them & the case in which they do not. To represent the universe, Euler's pairs of circles are here surrounded by a single large circle. In the corresponding section (§130) of the fourth edition, we are invited {6} at the end to compare this 'sevenfold scheme of class relations' with the 'sevenfold scheme of relations between propositions given in §84'. This parallel naturally did not become obvious to Keynes until he had enlarged the six propositional relations of the third edition to seven.

In Johnson I.iii.9, we are told a proposition p may be either (1) co-implicant to another proposition q, (2) super-implicant to it, (3) sub-implicant to it, (4) independent of it, (5) sub-opponent to it, (6) super-opponent to it, or (7) co-opponent to it. 'In traditional logic', he goes on 'the seven forms of relation are known respectively by the names equipollent, superaltern, subaltern, independent, sub-contrary, contrary, contradictory'. This is inaccurate; so far as I know, the term superaltern is innovation of Johnson's: in traditional logic (& in Keynes) both A and I, & both E & O, are called 'subalterns', the universal being 'subalternant' to its particulars & the particulars 'subalternate' to its universal. And in I.ix.7, Johnson reproduces Keynes's enlargement of Euler's diagrams (saying simply that they are 'due to Euler'), but with Keynes's outer circle replaced by a square. {7} [With regard to terms he says that P may be co-incident to it, sub-incident, super-incident, inter-sectant, super-remainder[?], sub-remaninder[?] or co-remainder to Q.] Neither Keynes nor Johnson refers explicitly to de Morgan, who had listed the same seven possibilities in his Formal Logic ch IV, but without diagrams; & as Keynes worked them and in connection with the diagrams it is possible that he had by that time forgotten the passage in de Morgan.