Letter from A.N. Prior to Alan Ross Anderson dated 7 Oct., 1955[1]

P.O. Box 1471

CANTERBURY UNIVERSITY COLLEGE

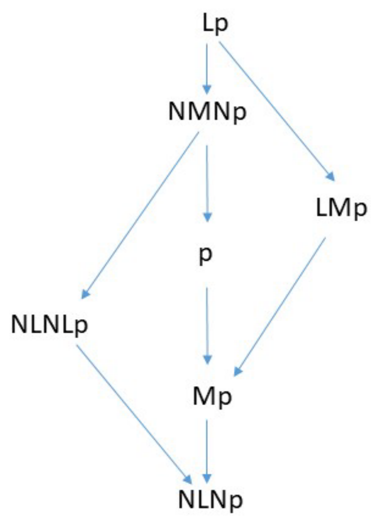
CHRISTCHURCH, C.1

NEW ZEALAND

7/10/55.

Dear Dr. Anderson,

Thank you for your second note about the 30-element matrix. I was relieved to find that the error in this matter was yours & not mine! And as far as I am concerned it has been a fruitful error, as it has pushed me towards a group of problems connected with Q which I am capable of doing something about myself.*[2][One thing left to consider: I won't prove Dugundji[3] &[4] other proofs about infinite matrices, including McKinsey's proof that there are an infinity of modalities in S2 (I worked out of this a tense-logical interpretation for Feys's [T]]. That proof that it has no finite matrix I am putting into an appendix along with it a proof I've just worked out that there are 14 modalities in Q – not the 14 of S4, but the following 7 & their negations :-



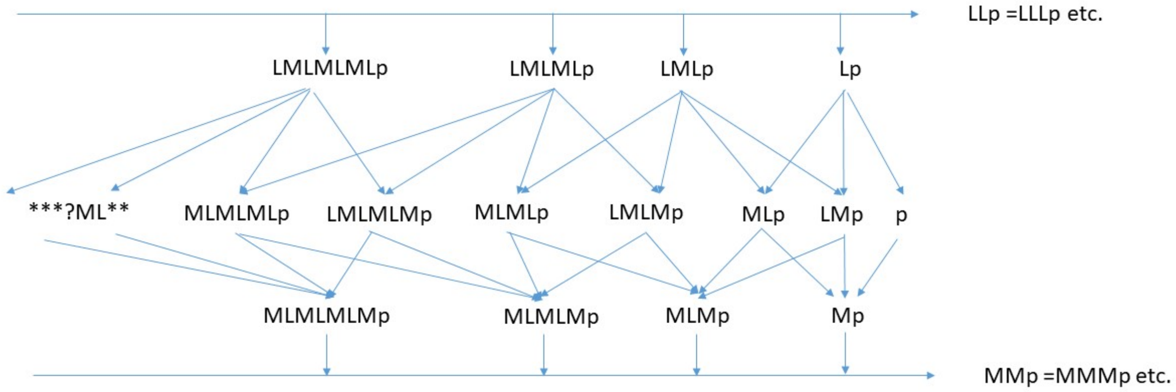
[1] This is a transcription of a handwritten letter kept in the Prior Collection at Bodleian Library, Box 1, Oxford. It has been edited by Max Cresswell, Adriane Rini and Peter Øhrstrøm. The letter is an aerogramme addressed to Dr. Alan Ross Anderson, 310 Linsly Hall, Yale University, New Haven, Connecticut, USA.

[2] Editors' note: The passage in [] is piece of vertical text in the margin with an indication that it should be inserted at this point.

[3] Editors' note: James Dugundji (1940) shows that none of S1–S5 has a finite characteristic matrix.

[4] Editors' note: Several words in this sentence are hard to decipher. We have tried to make the best sense of Prior's marginal note.

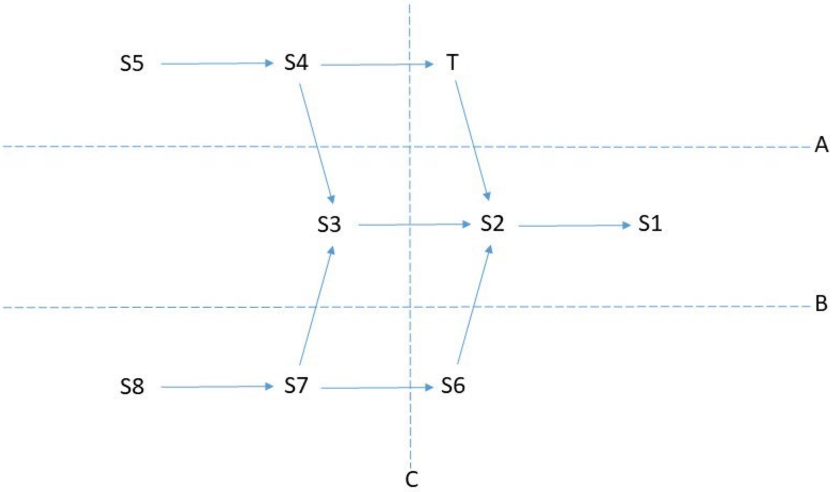
While working on this latter problem I was looking at the analogous problems for S6; & so far as I can see S6 has an infinity of modalities of which the affirmative ones go like this[5]:–



I have no proof of this, but I do not know of any reduction theorem in S6 which would enable me to collapse these any further. Do you know of anything against this conjecture?

{2} If there's nothing against it, it enables one to put the Lewis systems, plus Feys's system T, into a very neat picture, namely this one:–

[5] Editors' note: The left-most formula in the middle row of this diagram is hard to read, and we have transcribed it as best we can. We do, however, have a conjecture as to what Prior had in mind here. If you look at the formulae in the top-line of the diagram (except for Lp), you'll see that each has four formulae below it in the middle row (look especially at the case of LMLp). Each middle-row formula consists of either a sequence of LM's or a sequence of ML's. The two right-most in each of these four are obtained by deleting an L just before p, or by deleting an initial L. Each therefore has one fewer modal operator. The two left-most middle row formulae are obtained from the top-row formula by *adding* a modal operator, either M just before p or an initial M. They are therefore one operator longer that the top-row formula. So look now at the left-most formula of the top-row formulae. The two right-most of the middle-row formulae below it are obvious. But when you look at the two left-most arrows there is no middle-row formula below the left-most arrow and unclear writing below the other arrow. (It is possible that Prior realised that he did not have sufficient space, and that he crossed out what he had begun to write.) If the principle we have discerned were to be applied, then the missing left-most formula would be MLMLMLMLp, and the formula Prior appears to have crossed out should be LMLMLMLMp.



All systems above the line A contain the rule $\alpha \rightarrow L\alpha$ (& none below it do); all systems below the line B contain the law MMp (& with it NLLp), & none above it do; all systems to the right of the line C have an infinity of modalities (if my conjecture about S6 is right), & none to the left of it have.

Yours sincerely,

Arthur N. Prior