Letter from Alan Ross Anderson to A.N. Prior dated September 13, 1955[1]

September 13, 1955

Dear Professor Prior,

This letter is primarily to acknowledge receipt of the typescript of your lectures, for which I am much indebted to you. The typescript arrived today, along with two letters announcing the arrival. You guessed correctly that my new address is permanent (for a couple of years, at any rate).

I have not been to work on a formulation of Q until the last day or two, and I will now have to stop again, since classes begin today. As soon as things get organized, I hope to be able to turn to it again.

I have simply been tinkering around with the system, trying to get familiar with it. But I do have one result which may be of interest, namely, that there is a finite matrix for Q. The matrix[2] has 30 elements, each of which is a set of the strings which are elements in your infinite matrix.

Let [1, _2, _3] denote the set of strings which have 1 in first position, and have no 2's and no 3's. (Actually, of course, this is a unit set.) And [1,2,_3] is the set of strings which begin with 1, have at least one 2 (some place in the string) and no 3's. And so on. Then the elements are the following. (I will use underlined numerals to refer to the thirty elements.)

*1	[1,_2,_3]	5	[3,2,1]
*2	[1,2,_3]	6	[3,_2,1]
3	[1,_2,3]	7	[3,2,_1]
4	[1,2,3]	8	[3,_2,_1]

The starred elements are designated. The remaining twenty-two elements are disjuncts of pairs, triples, and quadruples of the first eight elements. I will use "v" for disjunction, and write "4v7" to mean the disjunct-set of [1,2,3] and [3,2,_1]. (Notice, incidentally, that sets 1 through 8 disjointly exhaust the universe.)

9	2v4	17	2v5v7	25	6v8
10	2v5	18	4v5v7	26	1v3v6
11	2v7	19	2v4v5v7	27	1v3v8

[1] This is a transcription of a letter kept in the Prior Collection at Bodleian Library, Oxford. It has been edited by Adriane Rini and Peter Øhrstrøm.
[2] Editors note: Anderson has typed 'matriz'.

12	4v5	20	1v3	28	1v6v8
13	4v7	21	1v6	29	3v6v8
14	5v7	22	1v8	30	1v3v6v8
15	2v4v5	23	3v6		
16	2v4v7	24	3v8		

[p. 2] Now using your original rules for manipulating the infinite matrix, one can construct in an obvious way matrices for K, N, L, and M, as applied to these thirty elements. But the four underlined operators are now to be conceived of as functions from sets to sets, rather than from strings to strings. To get the matrix for N, we say that Nx=y if and only if the (non-underlined) N maps members of x onto members of y. Thus N1=8, N2=7, N9=(14), N(11)=(11), and so on. And it turns out that for every set x, NNx=x, which means that N is an involution, just like N.

Similarly we construct matrices for L and M.

The case is a little more complicated for K, but the point is the same. Suppose we wish to calculate K(2)(5). [I am putting parentheses around the sets, so as to avoid ambiguity.]

First: No matter which strings from the sets 2 and 5 we take, the result of applying your K to the two strings will be a string with 3 in the first position, and at least one 2 somewhere in the string.

Second: Will the resulting string have any 1's? Well, it may, if it should happen that the 2-string and the 5-string both have 1 in the i-th position, for some i. But it may not; it might be that for every i, if the 2-string has 1 in the i-th position, then the 5-string has 2 in the i-th position, and if the 5-string has 1 in the i-th position, then the 2-string has 2 in the i-th position. In this case the resulting string would start with 3, and have nothing but 2's (and perhaps more 3's) following it.

So the resulting string may be either in [3, 2, 1] or in [3, 2, _1]. I.e., it may be either in 7 or in 5. I.e., it is in 14. So K(2)(5)=(14).

Similarly, we can compute a unique value for every pair x y of elements, and this gives us a table for K.

Working all this out in detail is a bore, and probably not worth the trouble. But it is evident that the two matrices satisfy the same formulae. A formula ϕ composed of variables and K, M, L, N is a law if its value for all assignments of strings to variables is a string with no 3's (i.e., is a string in 1 or in 2). And if a formula ϕ composed of variables and K, M, L, N, has the value 1 or 2 for every assignment of sets to variables, this would mean that no matter how one chose strings from the sets assigned to variables, one would wind up with a string in 1 or in 2, which would be a designated string in your infinite matrix. It shouldn't be too much trouble to dope out a rigorous proof for this.

The fact that the infinite matrix is equivalent to a finite one leads me to believe that it shouldn't be too much trouble to find an axiomatization of Q. What I will now do is read your ms as soon as possible, and then look into the axiomatization. (This probably within a month or so.)

Again, many thanks for the ms, which I look forward to reading.

Yours truly,

Alan Ross Anderson