

## Letter from Saul Kripke to A.N. Prior, Oct. 13, 1958<sup>1</sup>

Oct. 13, 1958

Dear Professor Prior,

Your letters have been received, although thus far your printed material has not. Incidentally, my address at Harvard can now be given more exactly as Weld 42, Harvard, Cambridge 38, Mass. (The zone numberings in Omaha shouldn't cause any difficulty on your printed matter; this change in numbering was made recently.) I don't know how often I'll be "hopping across the pond"—after all, I am only a college freshman—but when I do, I'll try and put Manchester on my itinerary.

Lemmon's matrices, like all such linear attempts, are faulty. In fact, his matrices for E2 and S2 verify  $CLC_{pp}CKMK_{pq}KMP_{Nq}Lp$ . However, I have found matrices of my own for S2 and E2 which work, using trees as before, but now permitting *endpoints* — sorts of Armageddons after which time stops. Then for E2 we evaluate every point except as in T; in an endpoint  $Lp$  is always "0" no matter what  $p$  is. (Note that two trees with endpoints are not necessarily alike in structure—in evaluating  $Apq$ , we consider only the common part of  $p$  and  $q$  as to structure.) S2 is like E2 except that the origin is evaluated as in T even if it is an endpoint.

Lemmon's matrix for S3 seems irreparable even if we use trees. It verifies  $MLLC_{pp}$ ; but what about S8, a consistent extension of S3?

You mention the L-modal system without functorial variables; did you know that this system should really be credited not to Lukasiewicz but to Curry—as far as priority goes? Curry first proposed this system in the last section of his A Theory of Formal Deducibility; there he had serious misconceptions about it, but he cleared these up in "The Elimination Theorem when Modality is Present", J.S.L. '52. There he gives the interpretation of  $Mp$  as  $CMp$  which you mention in your article on the British logicians' conference. I have succeeded in proving the elim. thm. for his Gentzen-like formulation, and hence the correctness of his other conjectures. As far as I know, no one seems to know about this—even Curry himself may not be aware of the connection with Lukasiewicz.

I think I ought to be able to prove the following to be an axiomatization of the strict implication part of S4, using your notation with respect to C and capitals— $CPCqP, Cpp, CCpqCCpCqrCpr$ , subst, det. The crucial step in a proof of equivalence with S4 would be the deduction theorem: If  $P_1, \dots, P_n, p \vdash q$ , then  $P_1, \dots, P_n \vdash Cpq$ .<sup>2</sup> I haven't quite worked out the details.<sup>3</sup>

Undoubtedly "trees" will be of some interest from the point of view of tense logic, although I haven't yet given much thought along this line. However, I am a little uncertain as to your own beliefs concerning the importance of tense logic. Is it your contention in Appendix A that a tenseless logic is really insufficient to represent the distinctions tense logic conveys? Do you think a tensed logic is needed for scientific discourse? I should think that, for scientific discourse a tenseless logic may be preferable. For example, in relativistic physics two events may be simultaneous to one observer but not to another, so that in tense logic *one* would say, "It is the case

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<sup>1</sup> Edited by Thomas Ploug and Peter Øhrstrøm. The letter is kept at Bodleian Library, Oxford. An earlier edition has been published in *Synthese* (2012) 188, p. 372 ff. The correspondence between Prior and Kripke has been discussed in Thomas Ploug & Peter Øhrstrøm: "Branching Time, Indeterminism and Tense Logic. Unveiling the Prior-Kripke letters", *Synthese* (2012) 188: 367-379.

<sup>2</sup> Turnstyles are missing in the original letter.

<sup>3</sup> Added in Prior's hand: "Barcan, Deduction Theorem etc., JSL XI (1946) p. 117".

that A, and it is the case that B,” while *another* (2nd) would say. “It is the case that A, it is not now the case that B, although it will be the case that B.” And if we accept indeterminism, the second might not even be sure that B will be the case. It thus seems that tenseless logic would be better equipped to handle these distinctions without contradiction, and without making propositions dangerously relative to the individual, rather than public property. (By saying that propositions are public property, I mean that they have the same meaning and truth-value to all observers. Maybe sameness of meaning is an exorbitant demand, but certainly sameness of truth-value is not.) And your arguments in the appendix by no means convince me that there are any real inadequacies in a tenseless logic. However, we do often use tense logic in ordinary discourse and it is of some interest to formalize it and investigate it, as you have done. The analogy with modality of course heightens our interest. The phrase “two possible times” meant that only events occurring in two times, this one and the one that actually turns out to be next, are called “possible”. In a tree matrix for T, there are infinitely many “possible times”, this moment and all the possibilities for the next. Thank you very much for your conjectural material on Q, etc. No comment as yet; when I have any, I will tell you.

Sincerely yours,  
Saul Kripke