Letter from J.J.C. Smart to A.N. Prior, November 15, 1953[1]

<div align="right">15.11.53</div>

Dear Arthur

Before getting down to brass tacks one quibble![2] (1) Surely Church's λ conversion isn't the same as combinatory logic. I know what the former is (though I don't understand it) because we have Church's monograph in the library. I'm still waiting for you to tell me what combinatory logic is! (2) When I talked about there being only two values for a truth operator of one argument that was a slip of the pen. What I meant was that Sp had only 2 possible values. (Of course S(p,q,r) has only two possible values too, so I must have been muddled.) Compare the double use of 'function' in maths. Sometimes by 'sin x' we mean 'λx. sin x' but sometimes the y such that y=sin x. But I withdraw quibble no2 as I certainly missed V and F, and only mentioned S and N. Half the trouble is is I've forgotten what [p.2] I wrote in the letter! Yes, my 'M' does seem to be only a special case of your 'T'. To revert to it

$$CpMq =_{Df} NCpq$$

By putting p = False it is obvious the def. leads to trouble.

  Now what rule of definition have we broken here? It ought to be possible to formulate it. What Hempel says on pp 18-19 of his 'Concept Formation in Empirical Science'[3] (reviewed by me in Phil. Review for July '53) is relevant. He says 'The introduction of certain kinds of nominal definitions into a given theoretical system is permissible only on condition that an appropriate nondefinitional sentence, which might be called its <u>justificatory sentence</u>, has been previously established. Thus, e.g., in Hilbert's axiomatization of Euclidean geometry, the line segment determined by two points, P_1 and P_2, is defined, in effect, as the class of points between P_1 and P_2 in the straight line through [p. 3] P_1 and P_2. This definition evidently presupposes that through any two points there exists exactly one straight line; and it is permissible only because this presupposition can be proved in Hilbert's system and thus can function as justificatory sentence for the definition! Hempel refers also to Peano's 'Les definitions mathématiques' in Bibliothéque du Congrés Internationale de Philosophie, Paris, III (1901) pp 279-88. (I doubt if we've got this in the library, though I haven't looked.) Peano gives an example[4] which obviously leads to trouble

$$\frac{x}{y} \, ? \, \frac{3}{n} = \, Df \, \frac{x+3}{y+n}$$

So
$$\frac{1}{2} \, ? \, \frac{2}{3} = \frac{3}{5} \quad \text{and} \quad \frac{2}{4} \, ? \, \frac{2}{3} = \frac{4}{7}$$

But
$$\frac{1}{2} = \frac{2}{4} \quad \text{so} \quad \frac{3}{5} = \frac{4}{7} \, !$$

What has gone wrong seems to be this.

$\dfrac{x+3}{y+n}$ is a function of 4 arguments.

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[1] Editor's note: The letter is in the Prior archive box 3 at the Bodleian Library in Oxford and has been transcribed and commented by David Jakobsen and Simon Graf.

[2] Editors' note: Smart has written 'one or two quibbles!' but has crossed over 'two'.

[3] Editors' note: Smart refers to Foundations of the Unity of Science. Volume II, no. 7: *Fundamentals of Concept Formation in Empirical Science* by Carl G. Hempel; International Encyclopedia of Unified Science, I and II.

[4] Editors' note: Smart in this letter shortens example with ex.

If $\left(\dfrac{x}{4} \ ? \ \dfrac{y}{3}\right)$ is a function of 4 arguments then '- ? –' is all one symbol. '$\dfrac{x}{y}$' here has no meaning by itself and so [p. 4] can't mean 'x÷y'. So Peano's example does not seem to be a good one. The geometrical one does seem more instructive. The definition makes use of the expression 'the straight line through P_1 and P_2' which clearly does presuppose the theorem 'here is one and only one straight line through P_1 and P_2' which is not universally true in certain sorts of non-Eucl. geometry. But reverting to

$$CpMq =_{Df} NCpq.$$

If p is false the Truth value of Cpq is independent of what q is. So it is easy to see why the definition of $CpMq =_{Df} NCpq$ leads to trouble in this case. To use Hempel's terminology I suppose the 'justification' theorem we would need is that Cpq and NCpr can always be given the same truth values by suitable choice of q for given r. And this theorem is not forthcoming.

But when is a justification theorem needed?

[5] I can't formulate any general rules for telling as this. Can you?

Thanks for the necessary existence thing. Did I ever send you my lecture on the existence of 'God'? In this I argue that 'logically necessary being' is self-contradicting like 'round square' simply because 'there exists a y' can never be a truth of logic. Your sentence 'For what cannot be thought of as attaching to a subject at all cannot be thought of as attaching necessarily to a subject'[5] seems to me to miss the point. For clearly 'exists' can be predicated of God, unicorns, lions, etc. (Even though there is a sense in which 'it isn't a predicate'!)

Now for the objection, I'm not sure that it will convince you for you have indeed made out a trickily plausible case! But I'd say that 'being red-all over and being not-red all over' only accidently, so to speak, entails non-exemplification.

What it primarily entails is [6] 'can't be exemplified'. But since 'can't be' entails isn't it also entails isn't exemplified. To prove exemplification won't do, I have a hunch, because 'can' doesn't entail 'is' in the way 'can't be' entails 'isn't'.

The question is: Does 'is red all over and not-red all over' entail 'can't be exemplified'? Or does 'can't be exemplified' simply mean '- entails isn't exemplified'?

But the general point remains. Logic can't tell you what is and what isn't but it can tell you what can be and can't be. But since 'can't' entails 'isn't' it can sometimes tell you what isn't. But it can't tell you what 'is'.

<div align="center">Yours Jack</div>

PS. Here is a pleasant puzzle I got from one of the mathematicians here. An aviator flying over darkest Africa is told that if he has to[6] (…)

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[5] Editors' note: The quotation can be found near verbatim in Prior, A.N., *Is Necessary Existence Possible?* Philosophy and Phenomenological Research Vol 15, No. 4 (1955): "For what is not rightly thought of as attaching to (being predicable of) a subject at all, cannot be thought of as attaching to a subject necessarily (or, of course, contingently)."
[6] Editors' note: The scanned document ends here and the rest of the PS is left out.