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Dear Professor Prior

Thank you for your letter and especially the information about Fine and Bull. I had earlier thought about some of the matters in my letter to you (in particular S5 with quantifiers), and reading your article stimulated me to prepare the enclosed abstracts, which I submitted for the January A.S.L. meetings. But now it appears that these results are not new. I hope that Fine and Bull will send me copies of their papers (you did not send me their addresses or I would write directly to them). [Marginal note reads: Both have written since this letter sent to the typist (Ed)]

I wonder if Fine has axiom sets with strong separation property I describe in "S5 with Multiple Possibility." I formulated my axiom in a slightly more complex way than necessary in the (vain) hope of achieving strong separability.

The theory "known to be decidable" is what Church calls singulary 2nd order logic with identity, not necessarily "uniform" in the sense of Quine. On re-reading my letter to you, I see that I expressed myself badly. The set of all valid sentences of the full theory is decidable, hence the set of valid sentences which are expressible in the sublanguage is also decidable. It is clear from the proof of decidability for the full theory that the same set of formulas can be axiomatized by the usual axioms for quantification theory (Tautologies, Distribution, Vacuous Quantification, Specification) for both individual and monadic predicate variables, plus the usual identity axioms ($\forall x\, x = x$, Leibniz' law). Since identity is definable with predicate variables ($\alpha = \beta$ for $\forall F(F\alpha \rightarrow F\beta)$), the identity formulas and axioms are superfluous. Thus since $\exists F[Fx \wedge \forall G(Gx \rightarrow \forall x(Fx \rightarrow Gx))]$ is valid, it must be provable just from the usual quantification axioms. And so it is. The F in question is, intuitively, singleton x ($\{x\}$), so take for the formula $F\alpha$, $\forall H(Hx \rightarrow Hx)$. Obviously, $\vdash \forall H(Hx \rightarrow Hx)$ and $\vdash \forall G(Gx \rightarrow \forall y(\forall H(Hx \rightarrow Hy) \rightarrow Gy))$, thus $\vdash \exists F[Fx \wedge \forall G(Gx \rightarrow \forall y(Fy \rightarrow Gy))]$, hence $\vdash \exists F[Fx \wedge \forall G(Gx \rightarrow \forall x(Fx \rightarrow Gx))]$. It surprised me that the corresponding axiom in S5 with quantifiers was independent.

It seems to me that you tend to think in terms of derivability while I tend to think in terms of validity, so possibly I have still not answered your question about the subtheory with the axiom (A)

(A) $\exists F[Fx \wedge \forall G(Gx \rightarrow \forall y(Fx \rightarrow Gx)]$.

Are you thinking of singulary second order logic sans identity and with only one individual variable (say 'x') ? The formulas of this last theory (call it V) are completely isomorphic to those of S5 with quantifiers. In fact the converse of the 2 of "S5 with Quantifiable Propositional Variables" will translate any formula (in the language of V into a formula in the language of S5Q. Thus my completeness proof for S5Q tells us that we can axiomatize V with the usual axioms plus (A), and my independence proof for my axiom (8) tells us that (A) is not derivable from the usual axioms so long as the derivation stays within the language of V.

If you still have reprints of "Egocentric Logic", I would like one, and other reprints also.

Should a visiting position here or at a nearby institution for a logician become available I will certainly mention Fine. (or is a student position e.g. teaching assistant appropriate?)

Sincerely,

David Kaplan

DK:bs

P.S. there has been a 3 week delay in typing this letter for which I apologize.

Editor’s note: The following formulae appear to have been written on the flap of the envelope. It’s not obviously Kaplan’s, and could be Prior’s since it is partly, though not entirely, in Polish notation.]

(∃δ)(∃γ) (nec.{δ . {γ . ~nec.}γ

(∃*p*)(∃*q*) (nec. *Np* . *Nq* . ~nec.*Nq*)

(∃δ)(∃γ) (nec. *F*δ . *F*γ ~nec.*F*γ) . Π*pN*δ*p* . *N*γ*p* . *N*Π*p*γ*p*

[Transcribed by M.J. Cresswell, from a version obtained by OCR from Kaplan’s typewritten letter. The Virtual Lab does not list any letters from Prior to Kaplan.]