Letter from Hans Kamp to A.N. Prior, March, 19, 1967

10949 Williams Ave.
Los Angeles 24.[1]

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Dear Professor Prior,

I have hardly any hope that you may forgive me my rudeness of never replying to your immediate and helpful letters which I received after you left Los Angeles. I hope I can explain (although I am aware that this is no real excuse!) how this came about and that it was not a matter of pure lack of courtesy. Soon after you left, a rather fascinating question in connection with the φ and ψ operators came up, and I believed that I would quickly arrive at a positive or negative answer to it. So I kept postponing answering your letters again and again, and because I felt worse and worse for not yet having written, I became more reluctant to write to you <u>without</u> the answer.

Now that the question has been settled, – and surprisingly enough in a positive way – let me attempt to explain it to you: {2}

The problem can briefly be stated as follows:

Assuming that time is linear, dense and without endpoints, can every <u>first order</u> <u>definable</u> term operator be <u>expressed in terms of</u> φ and ψ!

Here by a first order def. operator I understand any tense operator (of m places for arbitrary integer m!) there can be defined within linear, dense endpointless systems by formulae φ of predicate logic having only one free individual variable t_0 and m free non-predicate var's $G_1, ..., G_m$ The only non logical constant that may appear in the formula φ is a binary relation '<' (interpreted by the linear, dense order of the system!). Equality is included among the logical symbols of our pred. logic and may thus appear in φ. φ may contain additional bound ind. variables, but <u>no</u> <u>bound</u> set variables.

A tense operator N_φ is defined by such a formula φ in the sense in which

P is defined by $\mathrm{E}t_1(t_1 < t_2 \wedge G_1(t_1))$

and F -- -- -- -- $\mathrm{E}t_1(t_0 < t_1 \wedge G_1(t_1))$ {3}

and φ by $\mathrm{E}t_1(t_1 < \mathrm{t}_0 \wedge G_1(t_1) \wedge \forall t_2(t_1 < t_2 < t_0 \rightarrow G_2(t_2)))$

etc.

[1] Editors' note: This letter has been transcribed by Woosuk Park, Adriane Rini and David Jakobsen. It is located at the Bodleian Library Oxford in the Arthur Prior Collection, box 2. The original letter is kept at the Bodleian Library, Oxford, The Prior Collection, box 2.

N^m is <u>expressible</u> in terms of φ and ψ iff there is a formula η, built up from sentential variables $q_1, ..., q_m$, φ, ψ and the sentential connectives, s. th.

$N^m q_1 \ldots q_m \leftrightarrow \eta$ is valid in all linear, dense, endpointless tense structures.

The answer to the problem formulated on the previous page is indeed positive. The proof that I eventually found is quite complicated and very long (about 120 pgs.). I will send you a copy as soon as it will be in a form that is understandable to others than myself. It amounts to showing that every formula φ of the kind described on the previous pages is equivalent to a propositional function of formulae of a particular form – which correspond directly to formulae of the propositional tense logic with φ and ψ as primitive operators, whenever the symbol $<$ of our {4} predicate logic is interpreted by a linear dense order without endpoints.

Although the proof itself is absent and the definition of ‘first order definable tense operator’ and ‘expressible in given tense operators’, have only been indicated, I hope that these remarks may still convey some of the fundamental importance of this thm. As far as can see the notion of a 1st order definable tense operators is the most natural characterization of a tense operator in general. Under this assumption the theorem states that the tense logic with φ and ψ as primitives is, as it were, the “universal tense logic”, in which all temporal relationships among – or properties of – propositions that could be of interest for philosophy, can be expressed.*) [Author’s note: This claim however should be made with care, in uses of operators like ‘usually’ and ‘more often than not...’ and the like which might be considered tense operators on intuitive grounds, though they are not first order definable in the sense I indicated above.]

I showed in addition that φ and ψ cannot be defined in terms of any set of monadic (first order definable) tense operators, which destroys the hope that a universal tense logic with only monadic tense operators as primitives could be formed. (I include a proof of this second thm!) {5}

A recent result by Ehrenfeucht that the fragment of predicate logic which contains the “tense defining formulae” of pg 2) is decidable implies that the φ, ψ logic, and consequently all weaker prop. tense logics which have first order definable operators as primitives are equally decidable.

As far as axiomatisation is concerned I have not got very far yet. I worked on it during the summer for a while and found out that in view of the richness of the φ, ψ logic, the matter is very complicated. I will start again on this topic, with redoubled energy since the mentioned thm’s have pointed out the interest of this language.

I am inclined (and under pressure of Richard!) to change the φ and ψ into something else. S and U would be very appropriate, since natural readings for the two operators are:

‘*q* has been the case <u>since</u> *p*’

and ‘*q* will be the case <u>until</u> *p*’   resp.

The *S* would perhaps interfere with your *S*, where $Sp \leftrightarrow p$. But as in the φ, ψ logic, nor in your *P*, *F*, *G*, *H* logic {6} this last *S* plays a relevant part, this coincidence should be no real objection.

Since I started this letter I gave a talk about the principal results to the logic colloquium, the notes of which I include.

I have not yet looked closely into systems where time is supposed to have properties different from the rational (or real) numbers. But it seems to me that the theorem that everything is expressible in terms of Since and Until still will go through if one just assumes that time is linear and without endpoints (perhaps even the assumption that time is without endpoints might be dropped.)

As far as discrete time is concerned, it seems to me that the tense logic connected with this sort of time can be most naturally treated within the (*S*, *U*)-framework where time is supposed to be dense. For by means of the operators *S* and *U* we can express for an arbitrary proposition *p* the fact that it can be true only at discrete moments, […][2] by the formula

$GH\,(H'\neg p \wedge G'\neg p)$ ($\equiv D(p)$) where

G, *H* as usual and $U'p \leftrightarrow S(p \rightarrow p, p)$

$$G'p \leftrightarrow U(p \rightarrow p, p) \qquad \{7\}$$

This claim can be made precise in the following way.

Consider all formulae of tense propositional calculus with tense operators *S* and *U* in which the prop.'l variable *p* does not occur. We find on those formulae the function F by induction:

$f(q) = q$
$f(\zeta \vee \theta) = f(\zeta) \vee f(\theta)$
$f(\neg\zeta) = \neg f(\zeta)$

$f(S(\zeta, \theta)) = S(p_0 \wedge f(\zeta), p_0 \rightarrow f(\theta))$

$f(U(\zeta, \theta)) = U(p_0 \wedge f(\zeta), p_0 \rightarrow f(\theta))$

and finally $\eta^{(p)} = p \wedge Dp \rightarrow f(\eta)$

Then η holds in discrete time (or 'is a theorem of the tense logic of linear discrete time') iff $\eta^{(p)}$ holds in linear dense time.

This theorem can be proved recursively[3]

At the moment these are almost all the things I have to say. As soon as something new comes out of me, I will inform you. {8}

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[2] Editors' note: This word is unclear.
[3] Editors' note: This word is unclear, but seems to be what the author means.

I hope that everything is well with you and your family and that in particular your move to Oxford is in every sense positive. From what people told me I understand that one of the main advantages is the absence of administrative occupations.

My near future is still tied up with Los Angeles. I will teach here as a visiting professor in the Philosophy Department during the Spring quarter of 1968. I hope to complete my degree before then, but I expect that if Montague accepts the results I obtained in the domain of tense logic instead of the subject I originally proposed, this will not take too long any more. What will happen after next year is not yet clear. There is some possibility for an appointment in Berkeley but that is still very much up in the air.

Please give my best regards to Mrs. Prior and Ann.

Sincerely yours

Hans Kamp