Letter from Hans Kamp to A.N. Prior, March 31, 1967

L.A, March 31, 1967

Dear Professor Prior,[1]

Thank you very much for your three letters.

As far as the question raised in the last two is concerned, it seems to me that this problem is not entirely unambiguous.

As you remarked already the relation $x < y$ ('x precedes y') is not a linear ordering in circular structures.

For consider the usual axioms for linear ordering:

or' / or

1) $x \neq y \rightarrow x < y \vee y < x$

2) $x < y \wedge y < z \rightarrow x < z$

3) $x < y \rightarrow \neg (y < x)$

Obviously if we add to *or* the circularity connection:

4) $x < y \rightarrow y < z$

we obtain the theory:

$\exists x[\forall y(x = y) \wedge \neg (x < x)]$

which is not what we want.

But even if we delete 3) and add 4) to or'we obtain a theory in which $\forall x \forall y(x < y)$ holds, i.e. this predecessor relation is the universal binary relation. {2}

Indeed it is correct that circular orderings can be characterised only in terns of ternary relations/ see e.g. Lewis and Langford, Symbolic Logic chapters on Deduction and Deducibility).

As for the meaning of the operators S and U in such structures is concerned however, there seems to me to be a choice.

S(p, q) intuitively means:

(1) 'There was a moment at which p was true and for all moments <u>between</u> that moment and the present q was true.'

In linear systems it is appropriate to symbolise (1) as:

$$\exists t'(t' < t \wedge (p \wedge tt') \wedge \forall t''\big(t' < t'' < t \rightarrow (q \cup (?)t'')\big))$$

Where $t' < t'' < t$ is a convenient abbreviation of $t' < t'' \wedge t'' < t$.

Now one can stick to this formal representation of (1) in the underlying case. Then S(p, q) just means :

'There is a moment at which p was true and q is always true.'

However in this same rigid formal {3} sense every circular order is dense, since the reference

$\forall x \forall y(x < y \rightarrow \exists z(x < z < y))$

is clearly a theorem of $or' \cup$ (4).

If on the other hand we give the 'between' in (1) its full intuitive force, i.e. t'' between t' and t meaning « if we go in some given direction (let's call it 'the direction to the future'!) starting from

[1] Editors' note: This letter has been transcribed by Woosuk Park, Adriane Rini and David Jakobsen. It is located at the Bodleian Library Oxford in the Arthur Prior Collection, box 2.

t',then we will pass t'' before we reach t , » then in terms of this 'between' density is not [???] but asserts that between every two different elemts there is a third. But we can indeed express the property by postulating $S(p, q) \rightarrow S(S(p, q), q)$.

For suppose that some cycle contains two elements t', t such that t' preceeds t immediately, i.e. there is no t'' s.th. between (t', t'', t). Let p be true at t' only and q nowhere. Then $S(S(p, q), q)$ is false a *t. So* $S(p, q) \rightarrow S(S(p, q), q)$ is not universally true on that cycle.

I hope this is an answer to your question. As far as a title of my dissertation is concened, I really {4} had not thought about that yet. Since you ased the question however I thought I could as well commit myself already by now (not very committing) title :

'On Tense Logic and the Theory of Order'.

However this dissertation cannot be accepted before March of 1968 ; otherwise I would loose my fellowship for the first two quarters of the next academic year. This however will be the rather precise datum of my Ph.D. since I need it in order to hold a teachign psition during the last quarter.

Yours,

Hans K.