Letter from Hans Kamp to A.N. Prior, August 28, 1967

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Dear Professor Prior,

Thank you very much for the book you sent me (already more than two months ago) as well as the three papers that reached me via Dana.

I read your book by now completely. I think it is an excellent documentation for anyone who is interested in the field, without going into technical details that would perhaps deter many philosophers. I found the introductory and historical parts very useful for myself, since they provided me with a background information that I have lacked too long. (Perhaps I was not alert enough during the first few of the lectures that you gave in Los Angeles in '65, where probably much of this material must have been presented.) I also think that I have developed a better feeling for the way you operate and the particular problems you {2} are interested in. So for example your interest in axiom systems even as long as there are no completeness proofs for them of the sort as have been provided by other people for some of your systems later, i.e. relative to a semantics that involves concepts – like e.g. instants of time, – which one might want to do without entirely). Indeed if I understand your remarks on the metaphysical doubtfulness of the notion of an instant of time correctly, what you are mainly interested in, is what one might call an intrinsic characterization of the tautologies I, say, propositional tense logic, i.e. a characterization that does not refer to such extraneous concepts as for example moment.

A thing with which I still have difficulties is your treatment of world propositions. The [][2] point that the a certain part of ordinary language, which comprises the ordinary terms, {3} is perhaps more basic than the part in which one talks about moments and about certain facts being true at certain moments, I can understand. And I wonder if there is perhaps any linguistic evidence to this effect. One might perhaps know of some primitive languages that do already have a quite definite tense structure, but in which one cannot really speak about moments, – whereas no example is known of the converse, (i.e. a language in which one always expresses 'it rained' by 'there is a moment before now when it rains', or something like that). Perhaps the tensestructure of such a language would even be much more elaborate than that of say English, and perhaps 'moments' were just invented – indeed as a sort of logical constructs – at a certain stage in the development of language where the temporal relationships that had to be expressed became too complex. Anyway, this is just a {4} fantasy of the moment and my knowledge of the linguistics is not sufficient to determine whether it is complete nonsense or not. But assuming that this is indeed the structure of moments – i.e. that they are constructed on the basis of a tense structured language – why should each be characterized by a single proposition?

[1] Editors' note: This letter has been transcribed by Woosuk Park, Adriane Rini and David Jakobsen. It is located at the Bodleian Library Oxford in the Arthur Prior Collection, box 2.
[2] Editors' note: The word is unclear.

[3] Isn't that a rather strange metaphysical position itself? Even if a moment of time is just the unique 'thing' where certain collection of propositions is true, why should there be a particular proposition in that collection that is true at that point only. It is true that in ordinary speech we do have propositions of this kind, namely 'It is August 19, 1967', for example, but such propositions do not seem to belong to that more basic part of language that was there <u>before</u> the concept of a moment. It is this more general idea of identifying moments with {5} complete sets of sentences – rather than with individual sentences – that lies behind completeness proofs is the style of Dana, and we have seen how elegantly this idea works out there. (I agree, however, that as a matter of principle one has to be careful toward the temptation the technical elegance of a given formal procedure as evidence the philosophical sameness of the idea behind it.)

I am very sorry that I informed you so late about my results, especially now that I read in your book that you have considered the comparison of the expressive strength of the tense logics and the 'earlier-later' logic, and the reduction of the latter to a particular tense logic a quite important problem.

The papers of yours that I received from Dana, I found very interesting. Thank you very much for those too.

I have thought somewhat about the question you asked Dana in the accompanying letter but {6} see no direct way to reduce that problem to 'things that I know of. (Dana perhaps might be of more help, even though he more or less delegated the thing to me, saying that I would know more about it.) After that I can say now (but you probably know that already any way!) is that there is a close connection between your problem and the general problem: For which axiom systems of tense-logic can we prove completeness following the by now usual procedure: Show that every complete consistent set of formulae has a model. Constructing such a model by identifying moments with complete consistent sets of formulae, and then define between two such sets the relation '*a* earlier than *b*' by $\{P\varphi: \varphi \in a\} \in b$. Then every axiom of the system under consideration (Γ say, 'consistent' thus means 'consistent relative to Γ') is reflected by a property of the rela-{7}-tion *R*. In this way one has obtained completeness proofs for many systems (of modal as well as tense logic) and also one knows of many pairs $\langle \eta, \varphi \rangle$ of a tense logical formula η and a 1st order condition ϕ on a binary relation, that φ can be derived for the relation *R* above if $\eta \in \Gamma$, and that on the other hand η is valid in all structures $\langle A, R \rangle$, where *R* has the property φ.

Your problems however touch on a more general question than the ones just mentioned, and of which I do not know the answer.

Take any structure $\langle A, R \rangle$. Formulae of tense-logic have a natural interpretation in such structures only by interpreting propositional variables as subsets of *A* and the operator $P\varphi$ by:

'$P\varphi$ true at *a* iff there is a *b* s.th. *bRa* and φ true at *b*' and similarly for *Fp*). Then the assertion that η <u>is valid on $\langle A, R \rangle$</u>, i.e. true for all choices of subsets of *A* for the propositional variables in it, amounts to a purely universal 2nd order assertion about *R*. {8} So assume that the pair $\langle \eta, \varphi \rangle$ has the property:

(2) for all structures $\langle A, R \rangle$

$$\langle A, R \rangle \vDash \eta \text{ iff } \langle A, R \rangle \vDash \varphi$$

Is this the axiom system $K_t \cup \{\eta\}$ complete w.r.t φ i.e. is it the case for every formula ζ of tense logic that if for all structures $\langle A, R \rangle$ sth $\langle A, R \rangle \vDash \varphi$, $\langle A, R \rangle \vDash \zeta$,

[3] Editors' note: There is written in the margin at this point: "If more than one, can conjoin them. But if infinite no.?" The marginal note is written in red and appears to be Prior's hand.

then $K_t \cup \{\eta\} \vdash \zeta$?

This question can be generalized clearly to a stronger but simpler version:

If for all structures $\langle A, R\rangle$ if $\langle A, R\rangle \vDash \eta$ then $\langle A, R\rangle \vDash \zeta$, is then $K_t \cup \{\eta\} \vdash \zeta$? In other words we ask if K_t is a complete axiomatization of a certain fragment of the 2nd order theory of a binary relation with additional (unary) predicat<u>es</u>. I think this a very interesting question in itself and I am glad that your problem suggested it to me.

Assume that those last two questions (or at least the first one!) have been answered in the positive. {9}

Then your second con-

(7) jecture **[** that if $\varphi \to \psi$ is derivable in system I and if

(3) syst III $\cup \{\eta\} \vdash \varphi''$ and

(4) III $\cup \{\zeta\} \vdash \psi''$ and

(5) III $\cup \{\varphi''\} \vdash \eta$ and

(6) III $\cup \{\psi''\} \vdash \zeta$

(where ϕ'' and ψ'' are the appropriate translations of ϕ and ψ (i.e. with $L(a \to Fb)$ for bRa and $L(a \to q)$ for $G(a)$ and similarly φ' and ψ' are the translations appropriate for syst. *I*, only with *Taq* instead of $G(a)$ and *Uab* instead of *aRb*)

(8) then $K_t \cup \{\eta\} \vdash \zeta$ **]** follows immediately. For one easily verifies that (7) implies: for all $\langle A, R\rangle$, if $\langle A, R\rangle \vDash \varphi$ then $\langle A, R\rangle \vDash \psi$

and (3), (5) imply: for all $\langle A, R\rangle$

$\langle A, R\rangle \vDash \eta$ iff $\langle A, R\rangle \vDash \varphi$

and (4), (6) imply: for all $\langle A, R\rangle$

$\langle A, R\rangle \vDash \zeta$ iff $\langle A, R\rangle \vDash \psi$

Your first conjecture: 'if (3)-(6) and (8) then (7), depends on the completeness of your calculus syst I: it is true that {10}

(9) if for all $\langle A, R\rangle$, if $\langle A, R\rangle \vDash \varphi$[2]
then $\langle A, R\rangle \vDash \psi$, then
I $\cup \{\varphi'\} \vdash \psi'$?

This however is more than we need because it would be sufficient to have (9) for first those formulae φ, ψ which 'corresponds' to formulae η, ζ of tense logic ('correspond' in the sense of the pairs $\langle \eta, \varphi\rangle$ and $\langle \zeta, \psi\rangle$ above)

But again I can't know how to prove the first assertion and I am even more at a loss of how to prove the weaker second one, since that proof, if it is not to be automatically a proof of the 1st as well, should make essential use of the restriction to those φ and ψ that correspond to some η and ζ; I can't know of any model theoretical characterization of conditions on a binary relation which have this property, although I have been thinking lately about this particular question which, to my mind, is again also of interest from the point of view of ordinary {11} predicate logic.

Your last question: – whether if

$K_t \cup \{\eta\} \vdash \zeta$, then there are

φ, ψ so that (3)-(7) as above – again I cannot answer. Only I know that it could be true only if one drops the demand that φ and ψ should be 1st order conditions, since there are formulae of tense logic – like e.g.

$Pq \wedge PH\neg q \rightarrow P(H\neg q \wedge GPq)$

(The 'continuity axiom') which express conditions on R which are essentially not 1st order.

About my own work: Still no axioms for Since and Until, leave alone completeness. In continuation of the research that I mentioned to you already I discovered something quite unexpected: Contrary to what is the case for dense time, in discrete time Since and Until are expressible in terms of monadic operators, namely in terms {12} of the operator: 'ϕ was true in the past and at the moment before the last moment that it was true, it was false', its mirror image and further Dana's 'yesterday' and 'tomorrow'.

I do have some ideas on axiomatization; possibly one can obtain systems by first developing a semantic tableau method for the Since, Until logic and then analyzing the rules and replacing them by axioms. Completeness would result automatically in this way. But I should actually try it and that I have not done yet. (Shame!)

With best wishes and many thanks,

Yours,

Hans Kamp