Letter from Hans Kamp to A.N. Prior, April 12, 1967

Los Ang 12-4-'67

Dear Professor Prior,[1]

Herewith a set of notes that I wrote out for a seminar on pragmatics that Richard gives during this and the next quarters. They give a few suggestions of how to solve certain philosophical questions pertaining to sentences that involve tenses or other temporal notions. There is a treatment of 'now' as a 1-pl. operator; a treatment of dates, which is, I think, rather close to your idea on the subject and which I owe in large part to you, I recognise; and finally a development of operators like 'it was the case the day before' and 'it was the case yesterday' within a tense logic where time is dense. The latter idea is closely related to my idea of how to introduce metric in time (i.e. using propositional constants) and requires for the same reasons the operators Since and Until.*)[2]

I also did some work related to the problem that I mentioned in my letter of early September, (or late August; unfortunately I did {2} not keep a copy or that letter, so my references to it will be rather vague.) I think I mentioned in that letter that with every formula η of modal propositional logic one can associate a purely universal second order sentence φ_η with as its only non-logical constant a binary relation symbol R , which is true on a structure $<I, \underline{R}>$ whenever in every worldset indexed by I, where $\underline{R}$ is the alternative relation, η is true. (So e.g. if η is $\Box p \rightarrow p$, then φ_η =

$$(\forall P)(\forall i)((\forall j)(iRj \rightarrow P(j)) \rightarrow P(i))$$

Let D be the class of second order sentences that arise in this way. When doing completeness proofs for systems where we have a semantically defined notion of validity we want to know if a given conclusion C on the alternative relation can be expressed by a modal propositional formula η, i.e. if C is logically equivalent to φ_η for some η. One easily shows that this question is undecidable for arbitrary C (at least if one assumes that, if the answer for C is positive then we can find a particular η s.th. $\vDash C \leftrightarrow \varphi_\eta$. But that {3} is of course the situation in which we are interested). I found however a model theoretic characterization of D, i.e. a mathematical characterization of the following relation P_Δ between classes S of models of the form $<I, R>$ and single models C of this form which is defined by

$S\ P_\Delta \mathrm{C}$ iff for all $\varphi \in \Delta$, if for all $M \in S$ $M \vDash \varphi$, then $\mathrm{C} \vDash \varphi$

(So this is a preservation thm of the ordinary kind, like the one that says that a first order sentence is logically equivalent to a purely existential inference iff it is preserved under arbitrary extensions.)

A conjecture which would lead to a more interesting characterization of P_Δ and which would also have important other consequences is that following: Every $\varphi \in \Delta$ is logically equivalent to a first order sentence. I came upon this conjecture only a few days ago and have as yet no way to prove it. It sounds very surprising and unlikely bit still I was not able to come up with a counterexample. If you know one {4} I would be very glad to hear about it for then I would stop worrying about the problem, which, if true, is probably not very easy to prove.

If the conjecture is true, however, it would give an interesting difference between modal propositional logic in the strict sense (i.e. where we have only one operator $\Box$) and similar systems where we have several such operators; for even in tense logic where we consider just two such operators, *G* and *H*, and where moreover the alternative relations corresponding to

[1] Editors' note: This letter has been transcribed by Woosuk Park, Adriane Rini and David Jakobsen. It is located at the Bodleian Library Oxford in the Arthur Prior Collection, box 2.

[2] Kamp has written in the margin: "I also include a recent paper by Richard which includes the basic definitions used in these notes."

these operators are each other's converses, there are formulae of which the second order translations are equivalent to no first order sentence, viz. the continuity axiom.

Further the conjecture would probably help to give a proof of the general completeness theorem for modal propositional logic, which I think I proposed in my previous letter:

Let *A* be an axiom system for modal propositional logic, which has the {5} same rules as Lemmon's K and contains all the axioms of C. Then for every formula ζ of modal propositional logic we have:

$\vdash A\ \zeta$ iff for every structure $\langle I, \underline{R}\rangle$ if for all $\eta \in A$, $\langle I, \underline{R}\rangle \vDash \varphi_\eta$
then $\langle I, \underline{R}\rangle \vDash \varphi_5$.

I think a proof of this assertion would be very desirable for it would make the more and more complicated theorem of the first half of the Scott-Lemmon nook superfluous and also explain why such arguments as are used there do actually work.

Further all sorts of results concerning the decidability and axiomatizability of various systems of tense logic were recently obtained, some by Dana, some by myself. But I think it is better to send you later a chart that lists all these different results.

Actually it is not a particularly good moment for sending an account of achievements since many things are still up in the air. That I write you right now has indeed another reason.

David Kaplan got several inquiries {6} from other Universities about people that they could hire for next year and wants to propose me as a candidate. He thought that a recommendation from you might do a lot of good. Would you be willing to write on? David asked me to ask you if in that case you would send it directly to him *) [Editors' note: HK has included the address at the bottom of the letter, marked *): i.e. to Professor David Kaplan, Department of Philosophy, UCLA, Hilgard 405, Los Angeles, 90024] He then would send copies to the respective places.

I had hoped for a possibility to stay here, but for rather annoying reasons that seems to be out of the question. So at present I haven't the slightest idea where I might end up next year.

I hope everything with you is alright, and also with your family.

I still count on going back during next summer and hope very much to respond then to your invitation of last fall.

Logic year is exciting and instructive. Unfortunately I did not have enough time to spend on it thus far. I went to the seminar but did no research in these areas yet.

Best regards,

your Hans K