# Letter from A.N. Prior to Saul Kripke, September 10, 1958[1]

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10/9/58

Dear Kripke

Many thanks for your letter. It was odd that you should write to me at just this time; having heard of you from John Lemmon, I sent you only a few days ago some material that I thought would interest you. I posted it, though, to the address given in the JSL Dec. 1957, and hope it eventually reaches you. The JSL address is Omaha 3, while that on your letter is Omaha 32. Apart from any results of this mix-up, surface mail from here to the US takes some time, so don't expect to see it for a couple of months anyhow. Airmail, on the other hand, isn't bad; it won't, indeed, go from there to here and back between Sept. 3 and Sept. 10 (so I shall send this not to Nebraska but to Harvard); still, it has got here today.

I had better, while I'm at it, give you my own movements. I shall be here until November 26 of this year; thereafter on the high seas (on the Athenic, via Panama to England) for about a month; and thereafter at the Philosophy Dept., University of Manchester, Manchester 13, England. When you're hopping across the pond, as I've no doubt you'll be doing from time to time, I hope you won't leave Manchester out of your itinerary.

It was pointed out to me last year that my matrix for S4 wouldn't do, and I have publicly recanted on this point in a note in Philosophical Quarterly for July 1958 (that note, though, itself contains at least one error, that Parry's S4.5 is a system between S4 and S5—Lemmon has now proved its equivalence to S5). This result has now been worked into a whole heap of results authored mainly by Lemmon, Dummett, Geach and Hintikka (whom you'll no doubt be seeing in Harvard). The main ones I have at present from Lemmon are these: if QS = CMLpLMp (Obviously equivalent to your own ALMpLMNp), L = ALCLpLqLCLqLp, H = CKMpMqAMKpMqMKqMp (also used by you), G = CKpMLpLp, then S3 + G = S4 + G→ (but not =) S3 + L = S4 + L = S3 + H = S4 + H→ (but not =) S3 + QS = S4 + QS. Matrix verifying S4 + QS but not L is 16-valued Boolean for C-N with this for L:

| $p$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|-----|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|
| $Lp$ | 1 | 16 | 11 | 16 | 13 | 16 | 15 | 16 | 9 | 16 | 11 | 16 | 13 | 16 | 15 | 16 |

(1 designated). There is a general suspicion that S4 + L = D, i.e. Diodorean modality (characterized by my pseudo-S4 matrix), but so far as I know of no proof as yet. I have a proof in my note that if T is the Tarski–McKinsey transformation that correlates S4 with Heyting's calculus, then ├$\alpha$ by my matrix if and only if ├T$\alpha$ by that Gödel matrix in his 1932 thing; and Dummett has proved that this matrix is characteristic for Heyting + ├ ACpqCqp (which is TL).

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That my matrix for T is similarly faulty I did *not* know; nor did Lemmon when he pointed out the fault in my matrix for S4. He himself at that time suggested the following matrices for S2, his own E2, and S3: Use '$\alpha = 1m\ (0m)$' for '$\alpha$ takes the value 1 (0) in the m'th place', it being understood that for $m \geq 1$, $\alpha = 1m$ or $\alpha = 0m$ (and that $\alpha_{\_}=10$ and $_{\_}=00$). For this notation he goes

For S2: $L\alpha = 1m$ if and only if $\alpha = 1m$ and $\alpha = 1(m - 1)$; *i, 0i

For E2: $L\alpha = 1m$ if and only if $\alpha = 1m$ and $\alpha = 1(m - 1)$; *i,

For S3: $L\alpha = 0_1$;

$L\alpha = 1_2$ if and only if $(n)_{n>0}(\alpha = 1n)$

and for all $m > 2$, $L\alpha = 1m$ if and only if $(n)_{n \geq m}(\alpha = 1n)$ ; * i, 0i

I don't know whether these have faults similar to those you've found in mine.—I notice that while if QS is added to S3 only it gives the reduction thesis CMLpLMLp, it doesn't when added to T. About Q: The following are conjectures only, but may interest you:—Within QM for that portion 1Q which does not employ the operator L, Lemmon has found no law of QM that does not follow from p.c. + my rule M2 for S5 + the following modification of M1: $\vdash C\alpha\beta \rightarrow \vdash CM\alpha\beta$, if $\beta$ is fully modalised <u>and contains no variables not in $\alpha$</u>. And he has found no law of Q as a whole that doesn't follow from this lot together with the following further postulates involving L: the rule RL: $\vdash C\alpha\beta \rightarrow \vdash CL\alpha L\beta$, provided $\beta$ has no variables not in $\alpha$, and the axioms 1. CKLpLqLKpq, 2. CLpp, 3. CLpLLp, 4. CLNLNpMp, 5. CMqMCLpq. And I have proved these… (continued on other air-letter)[2]

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[2] The last part of the letter is missing.