Letter from A.N. Prior to Saul Kripke, Oct. 27, 1958[1]

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Dear Kripke

Thank you very much for your letter of the 13th.

Isn't the deduction theorem 'If $P_1, \ldots P_n$, p ├ q, then $P_1, \ldots P_n$ ├ Cpq' implicitly proved for S4 in Barcan's paper in JSL XI (1946) p. 117?—If your postulates are sufficient for C4, it follows easily that Lemmon's CqCpp, CCpqCCqrCpr, CCpCpqCpq are also sufficient. Your axioms follow easily from these, and I presume you can do the converse (I, I must confess, can only do it with your third axiom replaced by its lemma, i.e. the usual Frege CCpCqrCCpqCpr. Lemmon once had this sort of variant of a conjectural C5 base).

Yes, it is certainly my contention that you can do things with a tensed logic that you can't with a tenseless. I'd agree, though, that you get into trouble if you identify the earlier-later relation of relativistic physics with the earlier-later relation defined in the ordinary way in terms of tenses ('p earlier than q' as 'it is, has been or will be that KKPpNpq'); and would conclude that they are not in fact the same relation, though there is some sort of connection between them. As for what kind of logic 'quantified discourse' requires, that seems to me to depend on how much this term covers. There are areas of science within which certain questions arishing out of ordinary tense-distinctions aren't answerable, and in which 'it is better to use a language in which such questions cannot even be stated; and I take it that the moral of Special Relativity is that the theory of light-propagation is such an area.—In your paragraph about this you have a sentence beginning 'And if we accept indeterminism…', but I do not see how indeterminism can be expressed in a tenseless language at all. For indeterminism asserts a certain difference between the future and the past (that one has always APnpPnNp, but not always AFnpFnNp), which is not at all the same thing as a difference between the earlier and the later. And it seems to me that if any particular matter is at any time undetermined, that that is something that it is in itself and not just with respect to some observer—I cannot make any sense out of the idea of a thing being determined to one observer and undetermined to another (it can, of course, be within the reach of one agent's influence and not of another's; but that is a different matter). And since a thing can only be undetermined if it is not-yet-past, this also must be something which it is in itself and not just with respect to some observer.—I would say about this, in fact, what Gödel says about existence in Footnote 5 of his very important contribution to the Schilpp Einstein volume; though my conclusions are the opposite of his.

Thank you very much for drawing my attention to this t-model material in Curry.

Yours sincerely
Arthur N. Prior

PS: That the McKinsey matrix is not characteristic for T was apparently noted a couple of years ago in T. Smiley's (Cambridge) doctoral thesis. But Smiley hadn't a matrix that was characteristic for T (or for S4). An odd point I notice about the tense-logical interpretation of your 'trees' is that the passage of time, represented by the movement to new 'levels', is discrete. I don't know whether this feature is eliminable; I have sometimes myself wondered whether the notion of 'alternative futures' presupposes the discreteness of time.
A. N. P.