COMPUTATIONS AND SPECULATIONS[1]

by

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[1] Editors Note: This text has been edited, transcribed and annotated by Per Hasle. The text is kept in the Bodleian Library, Oxford, Box 8. The page numbers in the original text are clearly indicated in the following transcription. All underlinings are Prior's. The symbols used in this section which were not available on a standard typewriter, i.e. δ, Δ, Γ, Σ, ⊢, ⊣ are all added by hand in blue ink in the original manuscript. Footnotes fall into two groups: a) Prior's original notes, for which are given their original numbers in the manuscript; b) Editor's notes, which either describe Prior's handwritten editing of the text or represent comments or additional information given by the editor. Further information on the manuscript, e.g. on authorship, dating and its physical state, can be found at www.priorstudies.org, cf. Box 8.

SECTION VI[2]

MODAL LOGIC WITH VARIABLE FUNCTORS AND THE CONTINGENT CONSTANT ‘EVERYTHING THAT IS THE CASE’

We return in this section to versions of propositional calculus with two sorts of variables - the ordinary propositional variables, and variables standing for functors of propositional arguments. In the C-0-δ-p[3] and C-N-δ-p calculi mentioned in the introduction to Section,[4] the only constants used in constructing expressions[5]

[2] Editor’s note: It should be noted that this chapter VI according to the table of contents (see *Nachlass*) originally ran to page 135, but that only pages 109-122 can be found in box 8. The title of the chapter is identical with the title of Meredith and Prior’s later publication: Meredith, C. A., Prior, A. N. 1965. ‘Modal Logic with Functorial Variables and a Contingent Constant’, *Notre Dame Journal of Formal Logic*, vol. 6, 99-109. Clearly that paper built on the missing pages 123-135, however it must have to been to some extent reworked – in particular section 1 of the 1965-paper raises an objection by Prior to Meredith’s contingent constant n which must have been introduced later. Futher discussion of the state of the manuscript is found in the relevant description at https://research.prior.aau.dk/priorstudies/box/8 as well as some of the references given there, especially (Rybaříková and Hasle 2017) and (Copeland and Hasle 2018).
[3] Editor’s note: Instead of ‘0‘, the manuscript has here an ‘O‘. The same is the case in a number of formulas in the following pages. In the margin is scribbled – in red ink and Prior’s hand – *nought*, an instruction to the secretary (Miss P. Horne) to replace the ‘O‘ by ‘0‘. This instruction is also repeated in the following pages. Here and in the following, we simply use the correct ‘0‘, and further instances of this instruction are omitted.
[4] Editor’s note: The section number is missing in the manuscript.
[5] Editor’s note: With the subsequent text in the frame crossed out, it is clear that the reading should be continued from “substitutable for δ are...“ immediately after the frame.

Meredith also touches in this Royal Irish Academy paper on the δ-extension of the purely implicational fragment of the propositional calculus, and in recent years he has been occupied with similar extensions of that fragment of the classical propositional calculus in which the sole functor used is equivalence ('if and only if'). Some of his notes on this subject are given in the present volume.

2. The Modal Systems of Łukasiewicz and Meredith

In the above-mentioned C-0-δ-p and C-N-δ-p calculi the only constants used in constructing expressions[6]

substitutable for δ are two-valued truth-functors; and with the range of δ thus confined, the laws last quoted are very obviously true, at least if our truth-functional logic is two-valued. But Łukasiewicz produced during this period a calculus, retaining the axiom CδpCδNpδq, into which were introduced functors Δ and Γ which he interpreted as the modal 'It is possible that --' and 'It is necessary that -- ', respectively.[7] These functors were interdefinable in the usual ways (Γ as NΔ N or Δ as N ΓN, with axiom-sets

adjusted accordingly), and they and other functors definable in terms of them were treated as substitutable for δ. Most modal logicians find this 'Ł-modal' system counterintuitive in the extreme; e.g. it contains the laws CΔpΔΓp, 'Whatever could be true could be necessary', and CΓΔpΓp, 'What must be possible must be true',[8] and still worse, CΔpCΔNpΔq, 'If any proposition and its negation are each of them possible, then anything at all is possible', and CEpqCΔpΔq, 'If p and q have the same truth-value, then if p is possible so is q' (so that we could not have p and q both false, and p possible but q not). The third of these is of course a direct substitution in Meredith's axiom CδpCδNpδq, and the fourth a similar substitution in

[6] Editor’s note: In the manuscript, a frame has been added by hand around this text and the text was partially crossed out. It was obviously the intention to delete this text.

[7] Original note 30: Jan Łukasiewicz, 'A System of Modal Logic', Journal of Computing Systems, Vol.l, No.3 (1953), pp.111-149; 'Arithmetic and Modal Logic', ibid, Vol.l, No.4 (1954, pp.213-219; 'On a Controversial Problem of Aristotle's Modal Syllogistic', Dominican Studies VII (1954), pp.114-128; and the concluding chapters of the second edition of Aristotle's Syllogistic. In some of these items Łukasiewicz employs the more usual M and L instead of Δ and Γ.

[8] Strictly sepaking, the formula says 'What must be possible must be necessary', but this of course implies 'What must be possible must be true'.

the 'law of extensionality' CEpqCδpδq; Łukasiewicz having shown[9] that Meredith's s axiom is deductively equivalent to any axiom-set sufficient for the non-extended propositional calculus with the law of extensionality subjoined.

Meredith's answer to the Ł-modal system, contained in the first two notes reproduced in the present volume, was a system with δ-variables, but with the δ-less portion equivalent to Lewis's S5. This is the Lewis system in which assertions of necessity or possibility are themselves always

either necessary or impossible; i.e. the system allows for contingent truths, but not for contingent modal truths. This 'two-valued' character of all 'modalised' propositions in S5 is a feature of it which Meredith exploits very fully in these and other papers on modal logic. For example, although he cannot retain the laws Cδ0Cδ1p (Cδ0CδC00δp) and CEpqCδpδq in modal contexts, he can and does have laws which are very like them, namely Cδ0Cδ1δΓp (Cδ0CδC00δΓp) and CΓEpqCδpδq. The latter may be roughly read as 'Necessarily equivalent propositions are interchangeable'; and within the system this is reasonable, for while even necessarily equivalent propositions are not interchangeable in all contexts (e.g. even though '2 + 2 = 4' and '326 + 172 = 498' necessarily have the same truth-value, for both are necessarily true, it does not follow that if I see immediately that 2 + 2 = 4 then I see immediately that 326 + 172 = 498), they do seem interchangeable in all modal and truth-functional contexts, and these are the only ones provided in Meredith's system. As to the formula Cδ0Cδ1δΓp, in a modal system it is convenient to take 0 as a necessarily false proposition and 1 as a necessarily true one (it will be this in any case if defined as C00), contingent propositions being thought of as having 'values' intermediate between these, so that our

formula amounts to 'What holds equally with a necessarily false and with a necessarily true proposition as argument, will hold with any assertion of necessity as argument'. For in S5 any assertion of necessity is either necessarily true, and so necessarily equivalent to any given necessary truth, or necessarily false, and so necessarily equivalent to any given impossibility.

As to the details of Meredith's axiomatisation, his undefined constants are (material) implication, necessity, and the constant falsehood 0. Negation is definable in the usual way as the material implication of the false; Np = Cp0. (That this particular 'false' is also impossible

[9] Original note 31: In the first paper in the preceding note.

does not affect the force of this definition; in particular it does not make it equate Np not merely with the falsity but with the impossibility of p. For although what necessarily implies the impossible must itself be impossible, what materially implies it need not be so). And the system is equivalent to a set of postulates for S5 with C,0 and Γ as primitives and using substitution and detachment (for material implication) as sole rules, supplemented by the weakened law of extensionality CΓEpqCδpδq and substitution for δ-variables. In fact, however, Meredith compresses all this to three axioms, one of them being the formula Cδ0CδC00δΓp mentioned above.

-115-

(Previous axiomatisation of S5 with substitution and material detachment as sole rules,[10] but without δ-variables, were rather longer than this.) We may compare this with the replacement of the non-modalised propositional calculus (shortest single axiom in C and 0, Meredith's 21-letter CCCCCpqCr0stCC*pCrp)[11], plus CEpqCδpδq, by Cδ0CδC00δp.

3. The Wittgensteinian ‘World‘.[12]

It is a peculiarity of most modal systems, including both Łukasiewicz's and Lewis's S5, that their asserted formulae will all remain theorems if we equate both 'necessarily p' and 'possibly p' with the simple p; e.g. the laws CΓpp and CpΔp collapse under this interpretation to the law of identity Cpp. But if this worries us, there are a variety of ways in which modal logic may be redeemed from triviality notwithstanding. We might, for example, abandon the standard modal systems for some non-standard one which does not have the property just mentioned, e.g. Lewis's S6, which has as one of its axioms ΔΔp (but certainly not the simple p), or one of E.J. Lemmon's extensions of my own system Q, with

-116-

[10] Original note 32: See, e.g. Leo Simons, 'New axiomatizations of S3 and S4, Journal of Symbolic Logic Vol. XVIII (1953), pp.309-316. Simons's postulates here for S5 are improved upon in Alan Ross Anderson, 'Independent Axiom Schemata for S5'' Journal of Symbolic Logic Vol. XXI (1956), pp.255-6.

[11] Editor’s note: The ’*’ is this editor’s indication of a symbol inserted by hand, which cannot immediately be interpreted. It looks like a ’t’, but if it were a ’t’, surely it would have been typed. In any case, apart from this mystery symbol, the formula is only 18 letters long. Further studies in Meredith’s writings might well lead to the correct formula, of course.

[12] Editor’s note: In the manuscript, a frame has been added by hand around this text and the text was crossed out. It was obviously the intention to delete this text.

NΓp (but certainly not the plain Np) as an axiom.[13] Or we might, as Lewis does in one of his postulate sets,[14] introduce quantifiers binding particular variables, and such laws as ΣpKΔpΔNp, 'For some p, p is possible and so is not-p' (but ΣpKpNp, 'For some p, both p and not-p ', the 'collapse' of the foregoing, will certainly not do). What Łukasiewicz does is to have not only 'asserted' but 'rejected' formulae, i.e. formulae preceded by a sign[15] indicating that they are not theorems.[16] For example, he axiomatically 'rejects' CΔpp, or in some versions of his system CpΓp, but these equally with their asserted converses collapse to Cpp, which one certainly does not wish to reject. Meredith considers rejection too, but with his system it is complicated and difficult, and his principal device for removing any suggestion of triviality from his modal logic is the introduction of a contingently true propositional constant.

How, we might well ask, could any particular contingent proposition be logically interesting? In fact other writers

-117-

than Meredith have introduced into modal logic contingent constants which have some claim to be logically interesting. Thus Alan Ross Anderson has his 'sanction' or 'penalty' clause S, in terms of which he defines the 'deontic' notions of obligation, forbiddenness and permissibility; e.g. 'It is forbidden that p' is defined as 'p necessarily implies the Sanction'.[17] And it is an important theorem or axiom of Anderson's that the Sanction is avoidable - right action would be impossible if it were not - while morality would be somewhat vacuous if the Sanction were not at the same time possible. Another contingent constant, modelled on Anderson's, is Charles Hamblin's 'That would be very surprising', used to define a certain sort of probability.[18] But the constant used by Meredith is 'the world', in the sense of Proposition 1 of Wittgenstein's Tractatus, i.e. 'everything that is the case'. One can think of this as a conjunction of all truths, or perhaps of all 'atomic' truths. Meredith symbolises it simply as n, and gives axioms which bring out its special character.

In the first place, n is itself an axiom (it is a truth formulable in the system). But it is not necessary; that too

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[13] Original note 33: See A.N. Prior, 'Notes on a group of new modal systems', Logique et Analyse, 6-7 (April 1959), pp. 122-7.

[14] Original note 34: Lewis and Langford, Symbolic Logic (1932), p.492, postulate B9, and see more generally pp.178 ff. in that work.

[15] Editor's note: A few words are deleted after this, namely: "⊣, the reversal of the standard assertion sign ⊢".

[16] Editor's note: In hand is added: (cf. V.2).

[17] Original note 35: Alan Ross Anderson, The Formal Analysis of Normative Concepts, Technical Report No.2, U.S. Office of Naval Research Contract No.SAR/Nour-609 (16), 1956. See also A.N. Prior, Time and Modality (Oxford 1957) Appendix D, and 'Escapism: The Logical Basis of Ethics', in [Editor's note: The rest of the reference is missing. It should however be: *Essays in Moral Philosophy*, ed. by A. I. Melden, University of Washington Press, Seattle, 1958, pp. 135–146.]

[18] Original note 36: C.L. Hamblin, 'The Modal "Probably"', Mind Vol. LXVII, (April 1959), pp.234-240.

-118-

(or that its being necessary would imply anything whatever, CΓnp) is an axiom. (Intuitively it is not necessary because there are contingent truths; and any conjunction with contingent components, e.g. the conjunction of all truths, is itself contingent). An incidental consequence of n's being a theorem but Γn decidedly not one is that the 'rule of necessitation' (if α is a theorem so is Γα) only holds without exception for that part of Meredith's system in which n doesn't occur. Finally, the 'comprehensiveness' of n (its being everything that is the case) is expressed by the axiom CpΓCnp, 'Whatever is true is necessarily implied by n' (for it will be a conjunct of it).

These three axioms for n may be simply subjoined to those for C-Γ-0-δ-p, or alternatively we may take our constant impossible proposition 0 to be precisely the falsehood that n is necessary, Γn, in which case the six axioms may be condensed to five. Professor Augustus F. Bausch has pointed out[19] that we may similarly in an Andersonian 'deontic' logic define 0 as ΓS, and that definitions of this sort give a certain interest to modal logics with implication but without negation.[20]

-119-

The 8-valued matrix which shows the independence of CpΓCnp, in both axiomatisations, has some philosophical interest. The 2^m values of the matrices which are frequently used for independence proofs in modal logic are interpretable as distributions of the ordinary two truth-values in m 'possible worlds'. At least two possible worlds, and so at least four values in a matrix of this sort, are clearly required to differentiate between contingent and necessary truth; and at least three possible worlds, and so eight values, are needed to distinguish 'worlds' from other contingent propositions.[21] For a 'world' is true only in one possible world, itself (ordinary language fails us a little here), so that a contingency distinguishable from a world must be true in at least two worlds; and there must be a third world in which it is false, if it is to be a contingency at all. Writing 111 .... 11, 111 .... 10, etc. for the 2^m values 'true in all m worlds', 'true in all but the m th', etc., we may say that a 'world' is a 'value' which has a 1 in one and only one place; which makes it, as Meredith says, 'next to impossibility', the 'value' which

-120-

[19] Original note 37: In a letter of l4th June 1956. [Editor’s note: Regrettably, this letter is not found in the correspondence kept in the Bodleian library.]

[20] Editor’s note: This is followed by this deleted sentence: 'This is a subject to which Meredith devotes attention in later notes reproduced in this volume.'

[21] Editor’s note: This is followed by this deleted sentence: 'For a 'world' is needed to distinguish 'worlds' from other contingent propositions.'

has no 1 anywhere. Thus to the Wittgensteinian insight that the impossible is the most that one can say,[22] Meredith adds that the actual world is the most that one can truly say, and any world is a maximum that one can say without contradiction. (What is not contained in, implied by, a 'world', is inconsistent with it). The logic of 'worlds' generally is developed further in later sections.

Connected with the representation of modal concepts by matrices is their representation by formulae of quantification theory. That S5 has such a representation, and a very simple one, was early noted by Wajsberg.[23] Meredith's way of setting out this representation is by reinterpreting propositions as 'properties', i.e. monadic predicates, truth-functions of propositions as property-forming functions of properties, and Γp as the assertion that p is a property of every object, such assertions being themselves treated, as in Tarski on Truth,[24] as properties possessed either by all objects or by none. In Meredith's system with n, n is represented by the property of being identical with a selected object a, formulae which express properties of a as well as

formulae which express properties of all objects being taken as theorems. This is analogous to the use of matrices in which the value n, or 'true in n only', is designated as well as the value 'true in all worlds'. For the representation in quantification theory of systems weaker than S5, Meredith has suggested introducing an axiomatically defined 2-place predicate U, and defining (Lp)a as IIbCUabpb.[25] (For iterated modalities, systematic alphabetic adjustments are obviously required, e.g. (LLp)a = IIbCUab(Lp)b = IIbCUabIIcCUbcpc.) Different axioms for U give different modal systems.[26]

While improving upon Łukasiewicz's system in these different ways, Meredith was also interested in the more compact axiomatisation of that system itself, and observed that if we take H (= impossibility) as sole primitive constant beside C (defining Np as CpHp and Δp as NHp), Łukasiewicz's two asserted axioms CδpCδNpδq and CpΔq may be replaced by the single axiom CpCδδHpδq and his axiomatic rejections ⊣CΔpp and ⊣Δp by ⊣CpHHp and ⊣CHpp. (The substitutions δ/', q/HHp turn the asserted axiom into CpΔp,

[22] Original note 38: Tractatus 4.463: 'Contradiction fills the whole logical space and leaves no point to reality'.
[23] Original note 39: M. Wajsberg, 'Ein erweiterter Klassenkalkül, Monatshefte für Mathematik und Physik, Vol. 40 (1933), pp.113-126.
[24] Original note 40: Reproduced as VIII in Tarski's Logic, Semantics and Meta-Mathematics. The crucial pages are pp.189-195. See also my notice of this work in Mind, Vol.LXVI, No.263 (July 1957), p.408.
[25] Editor's note: Instead of the Γ used as symbol for the necessity operator elsewhere in this text, Prior here suddenly adopts, or returns to, the more standardly used L for necessity. While this is partly explicable by the fact that L is used for necessity in the associated reference (as well as by Łukasiewicz and elsewhere by Prior himself), it does fit with the general picture that this manuscript had still not undergone thorough proof-reading.
[26] Original note 41: For fuller details, see A.N. Prior, 'Possible Worlds', Philosophical Quarterly. [Editor's note: Philosophical Quarterly, vol. 12 (1962), pp. 36 – 43. The lacking details in Prior's note suggest that the paper had been accepted but not yet published at the time note 41 was written.]

and if for p we put any theorem θ detachment gives CδδHθδq, in which Hθ functions like 0 in Meredith's C-0-δ-p axiom Cδδ0δp).